Math 3210 **Exercise only** **Assignment #10**

1. Write an essay "[Interesting, Amazing, Weird, Funny] *Facts Known About Analytic Functions.*" (about 2 pages) You may be allowed to use it during the Final Exam.

2. Compose a True/False or multi-choise question about analytic functions and solve it. If I like it I may use it for our final exam.

3. Find the Residue $\text{Res}_{z=a}f(z)$ for given function $f(z)$ at point a.

   (a) $f(z) = z^{-17}\sinh(z^2)$, $a = 0$.

   (b) $f(z) = z^{14}\cot z^{-3}$, $a = 0$.

   (c) $f(z) = \dfrac{2\pi i \cosh z}{(z + z^4)}$, $a = 0$.

   (d) $f(z) = \dfrac{z^{3/4}}{z + 2i}$, $a = -2i$, $0 < \arg z < 2\pi$.

   (e) $f(z) = \dfrac{Log(z+1)}{(z^2+1)^2}$, $a = -i$.

   (f) $f(z) = \dfrac{e^{2z}}{\cos(z)}$, $a = \pm\pi/2$.

4. Evaluate each integral, if contours have negative orientation.

   (a) $\displaystyle\oint_{|z|=2} \frac{e^z\,dz}{z(z-1)}$.

   (b) $\displaystyle\oint_{|z|=2} \frac{dz}{z^3(z+3)}$.

   (c) $\displaystyle\oint_{|z+2|=3} \frac{dz}{z^3(z+3)}$.

   (d) $\displaystyle\oint_{|z|=2} \tan z\,dz$.

   (e) $\displaystyle\oint_{|z|=4} \frac{dz}{z(z+1)(z+2)(z+3)}$.

   (f) $\displaystyle\oint_{|z|=2} \frac{z^3(1-3z)\,dz}{(1+z)(1+2z^4)}$.

   (g) $\displaystyle\oint_{|z|=2} \frac{z^5\,dz}{1-z^3}$.

   (h) $\displaystyle\oint_{|z|=2} z^3 e^{1/z}\,dz$.

5. Let C_N be a positively oriented square centered at the origin with sides parallel to the coordinate axes, and of size $(2N+1)\pi$.

   1) Show that

   $$\oint_{C_N} \frac{dz}{z^2\sin z} = 2\pi i\left[\frac{1}{6} + 2\sum_{n=1}^{N}\frac{(-1)^n}{n^2\pi^2}\right].$$

   2) Find the sum $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$.

   3) Modify your method to find $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^4}$ and $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^6}$.

   4) Discuss whether the method works for all integer powers $s \geq 2$ in $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^s}$.