Math 3210 **Due Nov 16** **Assignment #8**

1. Outline a proof of Gauss' Mean Value Theorem.

2. Explain how Maximum Modulus Principle follows from Gauss' Mean Value Theorem.

3. Evaluate the contour integral.

   (a) $\oint_{|z|=3} \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)}\, dz$
   Hint: Use partial fractions and then Cauchy's Integral Formula.

   (b) $\oint_C \frac{e^{2z}}{(z+1)^4}\, dz$ where C is a rectangle with vertices at $\pm 2 \pm i$.

   (c) $\oint_{|z|=1} \frac{e^{2z}}{(z+1+i)^4}\, dz.$

   (d) $\oint_{|z|=2} \frac{z^5}{(z+i)^3}\, dz$

   (e) $\oint_{|z|=3} \frac{z^3}{(z+2i)^5}\, dz$

4. Let m, n be natural numbers, R be a positive real number and w be a complex number such that $|w| \neq R$. Evaluate

$$\oint_{|z|=R} \frac{z^n}{(z+w)^m}\, dz$$

5. Let $F(z)$ be entire function and $|F(z)| < a|z|$ for some real positive number a and for all $z \in C$. Show that $F(z) = bz$, where b is a complex number.

   Hint: show that $F''(z) = 0$.

6. Let $G(z)$ be entire function and $\mathrm{Re}G(z) < a$. Show that $G(z)$ =const.

   Hint: Show that $\exp(G(z))$ is const.

7. Prove the *Minimum Modulus Principle*: Let $f(z)$ be analytic continuous function in a closed bounded region D. Assume $f(z) \neq 0$ in D. Then $|f(z)|$ reaches its minimum value on the boundary of D, but not in the interior of D.

8. Evaluate by Cauchy's integral formula

$$\oint_{|z|=2004} \frac{e^z}{z}\, dz,$$

   and use this result to evaluate

$$\int_0^\pi e^{2004\cos t} \cos(2004 \sin t)\, dt.$$