Math 3210 **Due Nov 9** **Assignment #7**

1. Write the statement of the Cauchy-Goursat Theorem. Read the proof of the Cauchy-Goursat Theorem (p.144-149). Write down the main steps of the proof.
2. Write the statement of the Cauchy Integral Formula. Read and write down the proof of the Cauchy Integral Formula.
3. Evaluate the contour integral. Assume each contour to be positively oriented. Use Cauchy-Goursat Theorem or Cauchy Integral formula where applicable.

   (a) $\oint_{|z|=1} \tan z\, dz$

   (b) $\oint_{|z|=4} \frac{z}{z+3}\, dz$

   (c) $\oint_{|z|=2} \frac{z}{z+3}\, dz$

   (d) $\oint_{|z|=3} \frac{z}{(z+2)(z+4)}\, dz$

   (e) $\oint_{|z|=2} \cosh z\, dz$

   (f) $\oint_{|z|=2} \frac{\cosh z}{z}\, dz$

   (g) $\oint_{|z|=2} \mathrm{Log}(z-5)\, dz$

   (h) $\oint_{|z|=2} (z-1)^{-1}\mathrm{Log}(z-5)\, dz$

4. Let contour C be a pentagon with vertices at $-1; \pm 3i; 2 \pm 2i$. For all integer n find value of the integral
$$\int_C (z-1-i)^n dz.$$
5. Which of the following is true? Give a proof or a counterexample.

   A: Let C be a closed simple contour. If $\int_C f(z)\, dz = 0$ then $f(z)$ is analytic in the interior of C?

   B: Let $f(z)$ be continuous and $\int_C f(z)\, dz = 0$ for any closed simple contour C. Then $f(c)$ is analytic.
6. What is the geometrical meaning of expression $-i \int_C \bar{z}\, dz$, here C is a simple closed contour.
7. Write down and simplify the integral of $\exp(-z^2)$ along each side of a rectangle with vertices at $\pm a; \pm a + ib$. Use Cauchy-Goursat Theorem to show that
$$\int_0^a e^{-x^2} \cos(2bx)\, dx = e^{-b^2} \int_0^a e^{-x^2}\, dx + e^{-(a^2+b^2)} \int_0^b e^{y^2} \sin(2ay)\, dy.$$
   Use this result to evaluate $\int_{-\infty}^{\infty} e^{-x^2} \cos(2bx)\, dx$.