Math 3210 **Due Tue Nov 2** **Assignment #6**

1. Give an example (with explanaition) of
   (a) simple closed non-differentiable curve;
   (b) differentiable but not smooth arc;
   (c) Jordan curve.

2. Let
$$z_1(t) = R_0 e^{it}, \quad z_2(s) = \sqrt{R_0^2 - s^2} + is, \quad z_3(q) = q + i\sqrt{R_0^2 - q^2},$$
where $R_0 > 0$ is a constant and t, s, q are real parameters. When the interval for t is given, $z_1(t)$ describes a curve in the complex plane. Give an alternative description of the same curve in terms of either $z_2(s)$ or $z_3(q)$ (or both) and find corresponding interval(s) for s and q if
   (a) $\pi/4 < t < 2\pi/3$
   (b) $-\pi < t < -\pi/3$
   (c) $3\pi/4 < t < 5\pi/4$
   (d) $-\pi/2 < t < \pi$

3. Evaluate contour integral along curve $C = \{z(t), a \le t \le b\}$.
   (a) $\int_C z^{-2}\, dz$, $z(t) = 5e^{2it}$, $0 \le t \le \pi/4$
   (b) $\int_C \bar{z}^2\, dz$, $z(t) = e^{-it}$, $0 \le t \le 2\pi$
   (c) $\int_C \bar{z}^{-n}\, dz$, $z(t) = e^{ikt}$, $0 \le t \le 2\pi/k$, where n and $k \neq 0$ are integers.
   (d) $\displaystyle\int_C \frac{3+z}{z}\, dz$, $z(t) = \sqrt{9 - t^2} + it, 0 \le t \le 3$
   (e) $\int 2xy + i(y^2 - x^2) dz$ with respect to triangular contour with vertices at points $z = \pm i$ and $z = -1$.

4. Do the following without evaluating the integral.
   (a) Let $C = \{2e^{it}, 0 \le t \le \pi/2\}$. Show that $|\int_C (z^4 - 1)^{-1}\, dz| \le \pi/7$.
   (b) Let C be a line segment from $z = -i$ to $z = 1$. Show that $|\int_C z^{-6}\, dz| \le 8\sqrt{2}$
   (c) Let C be a circle of radius R. Show that
$$\left|\int_C \frac{\mathrm{Log} z}{z^4}\, dz\right| < 2\pi \frac{\ln R + \pi}{R^3}.$$

5. **Extra Points Problem** Show that condition
$$\frac{\partial f(z, \bar{z})}{\partial \bar{z}} = 0$$
is equivalent to the Cauchy-Riemann equations.