Math 3210 **Due Thur Oct 21** **Assignment #5**

1. Find *all values* and *the principal value* of the complex expression
   (a) $(-2)^{2/\pi}$
   (b) $(1+i)^{i-1}$
   (c) $(1-i\sqrt{3})^{5/2}$
   (d) This little poem is dedicated to the days when all formulae were written in words:
   i to the i times i to the i
   times e to the π – please simplify.

2. Given $\sin z = \dfrac{e^{iz}-e^{-iz}}{2i}$, $\cos z = \dfrac{e^{iz}+e^{-iz}}{2}$, $\sinh z = \dfrac{e^{z}-e^{-z}}{2}$, $\mathrm{cosh} z = \dfrac{e^{z}+e^{-z}}{2}$ $\tanh z = \dfrac{\mathrm{sinh} z}{\mathrm{cosh} z}$, verify that
   (a) $\sin(z+w) = \sin z \cos w + \cos z \sin w$
   (b) $|\sin z|^2 = \sin^2 x + \sinh^2 y$
   (c) $\sin(iz) = i\mathrm{sinh} z$
   (d) $(\tanh z)' = \mathrm{sech}^2 z$
   (e) $(\tanh z)' = -\mathrm{sech} z \tanh z$

3. Show that $2\mathrm{tanh}^{-1}(e^{i\theta}) = \log(i\cot(\theta/2))$.

4. Find derivative $w'(t)$ of the complex valued function w w.r.t. real variable t.
   (a) $w(t) = \cosh(\sin(3t) + it^2)$
   (b) $w(t) = (u(t) + iv(t))^n$, $n = 1, 2, 3...$

5. (a) Evaluate integral $\int_0^\pi e^{(a+ib)t}dt$, where a, b are real numbers and t is real variable.
   (b) Use result from (a) to find $\int_0^\pi e^{at}\sin nt$, where a is real and n is a natural number.
   (c) Confirm your result in (b) using integration by parts.

6. Envaluate improper integral
   (a) $\displaystyle\int_1^\infty \frac{(t+2i)^2}{t^4}\,dt$
   (b) $\displaystyle\int_0^\infty \frac{200+300i}{\sqrt{t}(1+t)}\,dt$
   (c) $\displaystyle\int_{-\infty}^\infty \frac{t}{1+t^4} + \frac{i}{1+t^2}\,dt$

7. **Extra Points Problem** Find simple algebraic condition for real numbers k such that the principal value of the expression $(ik)^{ik}$ is real? Give few (approximate) numerical values of such numbers k.