Math 3210 **Due Thur Oct 7** **Assignment #3**

Definition 1 Complex Derivative.

$$f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}, \quad z \in C.$$

Definition 2 Continuity of complex function at point z_0.

$$\lim_{z \to z_0} f(z) = f(z_0).$$

1. Using definition, find derivative of z^n, for
   (a) any positive integer n;
   (b) any negative integer n, if $z \neq 0$.

2. Show using definitions only that $f(z) = \bar{z}$ is continuous but not differentiable at any point of the complex plane.

3. Find complex derivative using differentiation rules
   (a) $(-4z^3 + i)^9$
   (b) $\left(\frac{iz-1}{z^2+1}\right)^4$

4. Let $f(z) = u(r,\theta) + iv(r,\theta)$ be analytic. Derive formula

$$f'(z) = -i(u'_\theta + iv'_\theta)/z$$

   from the formula $f'(z) = e^{-i\theta}(u'_r + iv'_r)$ using Cauchy-Riemann equations in polar form.

5. Use Cauchy-Riemann equation in the appropriate form to determine where in the complex plane the following function is analytic. Find the derivative if it exists.
   (a) $f(z) = (\bar{z} + 2)^2$
   (b) $f(z) = \text{Im}z$
   (c) $f(z) = z^{-3}$, $z \neq 0$
   (d) $\sqrt{r}e^{i\theta/2}$, $r > 0$, $-\pi < \theta < \pi$
   (e) $2x + ixy^2$
   (f) $e^{\bar{z}}$
   (g) $r\cos\theta + ir$
   (h) $(e^{2x}\cos 2y + ie^{2x}\sin 2y)^{-1}$
   (i) $r^4 \sin 4\theta - ir^4 \cos 4\theta$
   (j) $\bar{z}^3$

6. **Extra Points Problem** What is the flaw in the following argument?

$$e^{i\theta} = \left(e^{i\theta}\right)^{2\pi/2\pi} = \left(e^{2\pi i}\right)^{\theta/2\pi} = 1^{\theta/2\pi} = 1.$$