Math 3210 **Due Thur Sept 30** **Assignment #2**

1. Find domain of the following functions and rewrite them in the form $u(x, y) + iv(x, y)$
   (a) $\dfrac{1}{z\bar{z} - 5}$
   (b) $\dfrac{1}{z + i}$
   (c) $z + \dfrac{1}{z}$
   (d) $\bar{z} + \bar{z}^3$
2. Use de Moivre Identity to show that for any natural n

$$\cos(n\theta) = \mathrm{Re} \sum_{k=0}^{n} \cos^{n-k}\theta \sin^k \theta \frac{i^k n!}{(n-k)!k!}$$

   Apply the formula in the following
   (a) Find $\cos(8\theta)$ if $\cos\theta = 0.3$.
   (b) Let n be even. Show that

$$\cos(n\theta) = \sum_{m=0}^{n/2} \cos^{n-2m}\theta(1 - \cos^2\theta)^m(-1)^m \frac{n!}{(n-2m)!(2m)!}$$

3. Rewrite in terms of z and $\bar{z}$ and simplify
   (a) $x^3 + y^3$
   (b) $ix + \dfrac{x}{x^2 + y^2} + y + \dfrac{iy}{x^2 + y^2}$
4. Let $f(z) = (z + i)^{-1}$. Find
   (a) $f(1/z)$
   (b) $f(f(z))$
   (c) $f(z + i)$
5. Let $w = z^2$.
   (a) Sketch domain in the z-plane which has rectangular image $-3 \leq \mathrm{Re}w \leq -2$, $2 \leq \mathrm{Im}w \leq 3$
   (b) Sketch domain in the z-plane which has image $\mathrm{Re}w > 0$ and $\mathrm{Im}w < 0$
6. Let $w = e^z$, $z = x + iy$.
   (a) What is the image of line $x = 1$
   (b) What is the image of line $y = \pi/4$
   (c) Sketch the image of $\ln 2 \leq x \leq \ln 3$, $-\pi/4 < y < \pi/4$
   (d) Sketch domain in the z-plane which has image $\mathrm{Re}w > 0$ and $\mathrm{Im}w < 0$
7. **Extra Points Problem** Show that for any integer n

$$\left(\frac{1 + i\tan\theta}{1 - i\tan\theta}\right)^n = \frac{1 + i\tan(n\theta)}{1 - i\tan(n\theta)}$$