

1. Write an essay "[Interesting, Amazing, Weird, Funny] *Facts Known About Analytic Functions.*" (about 2 pages)

This question will not be marked but you may be allowed to use it during the Final Exam.

2. Pretend that you are Cauchy/Riemann/Liouville/Gauss/Euler/Argand/De Moivre/Hamilton/Morera/Laplace/Mkondra/Goursat/Taylor/Laurent and type your favorite formula on my Message Board. <http://www.math.mun.ca/mkondra/okno/index.php> Please, select Math 3210

3. Compose a True/False or multi-choise question about analytic functions and solve it. If I like it I may use it for our final exam.

4. Find the Residue $\text{Res}_{z=a} f(z)$ for given function $f(z)$ at point a .

(a) $f(z) = z^7 \cos(z^{-2}), a = 0.$

(b) $f(z) = z^{-3} \cot z, a = 0.$

(c) $f(z) = \frac{\sinh z}{z^4(1 - z^2)}, a = 0.$

(d) $f(z) = \frac{z^{1/4}}{z + 2}, a = -2, 0 < \arg z < 2\pi.$

(e) $f(z) = \frac{\text{Log} z}{(z^2 + 1)^2}, a = i.$

5. Evaluate each integral, if contours have negative orientation.

(a) $\oint_{|z|=2} \frac{e^z dz}{z(z-1)}.$

(b) $\oint_{|z|=2} \frac{dz}{z^3(z+3)}.$

(c) $\oint_{|z+2|=3} \frac{dz}{z^3(z+3)}.$

(d) $\oint_{|z|=2} \tan z dz.$

(e) $\oint_{|z|=4} \frac{dz}{z(z+1)(z+2)(z+3)}.$

(f) $\oint_{|z|=2} \frac{z^3(1-3z) dz}{(1+z)(1+2z^4)}.$

(g) $\oint_{|z|=2} \frac{z^5 dz}{1 - z^3}.$

(h) $\oint_{|z|=2} z^3 e^{1/z} dz.$

6. Is it true or false that for any $R > 0$

$$\oint_{|z|=R} \exp(z + z^{-1}) dz = 2\pi i \sum_{n=0}^{\infty} \frac{1}{n!(n+1)!}.$$

7. Let C_N be a positively oriented square centered at the origin with sides parallel to the coordinate axes, and of size $(2N+1)\pi$. Show that

$$\oint_{C_N} \frac{dz}{z^2 \sin z} = 2\pi i \left[\frac{1}{6} + 2 \sum_{n=1}^N \frac{(-1)^n}{n^2 \pi^2} \right].$$

Find the sum $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}.$