**Due Fri Apr 2, 2004**

# Math 2000: Assignment #9, Winter 2004- Answers

# I. Evaluate the integral. Sketch the region of intgeration.

1. $\int_0^2 \int_0^x (1+x^2)^{1/3}\, dydx = \frac{3}{8}(5^{4/3}-1),$ 2. $\int_1^2 \int_y^{y^3} e^{x/y}\, dxdy = \frac{e^4-4e}{2},$

# II. Sketch the region of intgeration. Evaluate by reversing the order of integration.

1. $\int_0^1 \int_{3y}^3 \sin(x^2)\, dxdy = \frac{1-\cos 9}{6},$ 2. $\int_0^8 \int_{y^{1/3}}^2 e^{x^4}\, dxdy = \frac{e^{16}-1}{4},$

# III. Evaluate integral using polar coordinates

a) $\int\int_D \tan(x^2+y^2)\, dA = \frac{\pi}{2}\ln|\cos(\pi^2/16)|$ $D = \{(x,y)|x \geq 0, x^2+y^2 \leq \pi^2/16\}$

b) $\int\int_D \sec^2(x^2+y^2)\, dA = \frac{\pi \tan 2}{4},$ $D = \{(x,y)|x \leq 0, y \geq 0, x^2+y^2 \leq 2\}$

# IV. Evaluate integral by converting to polar coordinates

1. $\int_0^1 \int_0^{\sqrt{1-x^2}} x^2y^2\, dydx = \frac{\pi}{96},$ 2. $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{x^2+y^2}\, dxdy = \frac{\pi}{2}(e^4-1),$

# V. Use polar coordinates to find the volume.

a) bounded by cone $z = (x^2+y^2)^{1/2}$ and the plane $z = 4$. Volume $=\frac{64\pi}{3}$.

b) bounded by the paraboloids $z = x^2+y^2-4$ and cone $z = 2-(x^2+y^2)^{1/2}$. Volume $=\frac{32\pi}{3}$.

c) bounded by the upper half of the sphere $x^2+y^2+z^2 = 1$ and paraboloid $z = 1-x^2-y^2$. Volume $=\frac{\pi}{6}$

**BONUS** problem.

a) Evaluate $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}\, dxdy$

b) Use your result in (a) to calculate $\int_{-\infty}^{\infty} e^{-x^2}\, dx$ friequently used in statistics.