

Due Fri Apr 2, 2004

Mathematics 2000: Assignment #9, Winter 2004

I. Evaluate the integral. Sketch the region of integration.

$$1. \int_0^2 \int_0^x (1+x^2)^{1/3} dy dx, \quad 2. \int_1^2 \int_y^{y^3} e^{x/y} dx dy,$$

II. Sketch the region of integration. Evaluate by reversing the order of integration.

$$1. \int_0^1 \int_{3y}^3 \sin(x^2) dx dy, \quad 2. \int_0^8 \int_{y^{1/3}}^2 e^{x^4} dx dy,$$

III. Evaluate integral using polar coordinates

$$\begin{aligned} \text{a) } & \int \int_D \tan(x^2 + y^2) dA \quad D = \{(x, y) | x \geq 0, x^2 + y^2 \leq \pi^2/16\} \\ \text{b) } & \int \int_D \sec^2(x^2 + y^2) dA, \quad D = \{(x, y) | x \leq 0, y \geq 0, x^2 + y^2 \leq 2\} \end{aligned}$$

IV. Evaluate integral by converting to polar coordinates

$$1. \int_0^1 \int_0^{\sqrt{1-x^2}} x^2 y^2 dy dx, \quad 2. \int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{x^2+y^2} dx dy,$$

V. Use polar coordinates to find the volume.

- a) bounded by cone $z = (x^2 + y^2)^{1/2}$ and the plane $z = 4$.
- b) bounded by the paraboloids $z = x^2 + y^2 - 4$ and cone $z = 2 - (x^2 + y^2)^{1/2}$.
- c) bounded by the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and paraboloid $z = 1 - x^2 - y^2$

BONUS problem.

- a) Evaluate $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2-y^2} dx dy$
- b) Use your result in (a) to calculate $\int_{-\infty}^{\infty} e^{-x^2} dx$ frequently used in statistics.