MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

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**Mathematics 2000** **Assignment 8 -Answers** **Winter 2004**

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**Due March 24**

1. Use the Chain Rule to find $\frac{dv}{dt}$, given $v = \tan(2x + 3y)$ with $x = t^3$ and $y = t^{1/3}$.

   Find value $\frac{dv}{dt}$ at $t = 1$.

   $\frac{dv}{dt}(t) = (6t^2 + t^{-2/3})\sec^2(2t^3 + 3t^{1/3})$

   $\frac{dv}{dt}(1) = 7\sec^2(5)$

2. Use the Chain Rule to find $\frac{\partial v}{\partial s}$ and $\frac{\partial v}{\partial t}$, given $v = x(y + 2z)^{1/2}$, with $x = s^2/t$, $y = t^2 + s^2$ and $z = -st$.

   Find $\frac{\partial v}{\partial s}$ at $s = 1$, $t = 2$.

   $\frac{\partial v}{\partial s}(s,t) = (t^2 + s^2 - 2st)^{1/2} 2s/t + (s^3/t)(t^2 + s^2 - 2st)^{-1/2} - (s^2)(t^2 + s^2 - 2st)^{-1/2}$

   $\frac{\partial v}{\partial s}(1,2) = \frac{1}{2}$

3. Differentiate implicitly to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$:

   (a) $x^3 + y^3 + z^3 = xy + yz + xz$

   $$z_x = \frac{y + z - 3x^2}{3z^2 - y - x}$$

   $$z_y = \frac{x + z - 3y^2}{3z^2 - y - x}$$

   (b) $x - y = \arctan(yz)$

   $$z_x = \frac{1 + y^2 z^2}{y}$$

   $$z_y = -\frac{1 + y^2 z^2 + z}{y}$$

   (c) $xyz = \sin(x + 2y + 3z)$.

   $$z_x = \frac{\cos(x + 2y + 3z) - yz}{xy - 3\cos(x + 2y + 3z)}$$

   $$z_y = \frac{2\cos(x + 2y + 3z) - xz}{xy - 3\cos(x + 2y + 3z)}$$

4. Determine whether there is a relative maximum, a relative minimum, a saddle point, or insufficient information to determine the nature of the function $f(x, y)$ at the critical point

   (a) $f_{xx}(x_0, y_0) = 3,\ f_{yy}(x_0, y_0) = -8,\ f_{xy}(x_0, y_0) = 3$. Saddle Point

   (b) $f_{xx}(x_0, y_0) = 3,\ f_{yy}(x_0, y_0) = 8,\ f_{xy}(x_0, y_0) = 3$. Local Minimum

   (c) $f_{xx}(x_0, y_0) = -2,\ f_{yy}(x_0, y_0) = -8,\ f_{xy}(x_0, y_0) = -4$. Inconclusive

   (d) $f_{xx}(x_0, y_0) = -2,\ f_{yy}(x_0, y_0) = -8,\ f_{xy}(x_0, y_0) = 2$. Local Maximum

5. Find and classify critical points.

   (a) $f(x, y) = 10 - x^2 - y^2 + xy$
   The function has maximum at point $x = y = 0$.

   (b) $f(x, y) = 10 + x^3 + y^3 - 3xy$
   The function has local minimum at $x = y = 1$, and saddle point at $x = y = 0$

6. Calculate the iterated integral.

a) $\displaystyle\int_0^2 \int_0^4 (1 + x + 2y)\, dxdy = 40$

b) $\displaystyle\int_0^2 \int_0^4 (1 + x + 2y)\, dydx = 48$

c) $\displaystyle\int_1^2 \int_0^{\pi/2} x \sin(xy)\, dydx = 1 + \frac{2}{\pi}$

d) $\displaystyle\int_0^{\ln 2} \int_0^{\ln 3} ye^{x+y^2}\, dxdy = e^{(\ln 2)^2} - 1 = 2^{\ln 2} - 1$

e) $\displaystyle\int_1^2 \int_2^4 \frac{x^2}{y}\, dxdy = \frac{56 \ln 2}{3}$

f) $\displaystyle\int_0^1 \int_0^1 \frac{x^2 y}{\sqrt{x^3 + y^2 + 10}}\, dydx = \frac{2}{9}(12^{3/2} - 2(11)^{3/2} + 10^{3/2})$

g) $\displaystyle\int_0^{0.5} \int_0^1 \frac{1 + x^2}{1 - y^2}\, dxdy = \frac{2}{3} \ln 3$

h) $\displaystyle\int_0^{\pi} \int_0^{\pi/2} \cos(x - y)\, dydx = 2$