Due March 17 2004

Mathematics 2000: Assignment #7, Winter 2004

1. For the following functions find domain, sketch contor map, sketch the graph of the function, name it.

   (a) $F(x,y) = 1 - \sqrt{x^2+4y^2}$
   (b) $F(x,y) = 1 + \sqrt{16 - x^2 - y^2}$
   (c) $F(x,y) = 4x^2 + y^2 - 2$
   (d) $F(x,y) = -\sqrt{1 - x^2 - 4y^2}$
   (e) $F(x,y) = 1 + x$
   (f) $F(x,y) = \frac{x^2}{4} - 4y^2$

2. If the limit exists, find it. Otherwise show that it doesn't exist.

   (a) $\lim_{(x,y)\to(0,0)} 2\cos(x^3) - 3e^x$
   (b) $\lim_{(x,y)\to(1,1)} \dfrac{\sin(x^2+y^2)}{x^2+y^2}$
   (c) $\lim_{(x,y)\to(0,0)} \dfrac{\sin(2x^2+3y^2)}{2x^2+3y^2}$
   (d) $\lim_{(x,y)\to(0,0)} \dfrac{2x^2-3y^2}{2x^2+3y^2}$
   (e) $\lim_{(x,y)\to(0,0)} \dfrac{x^2y}{x^4+y^2}$
   (f) $\lim_{(x,y)\to(0,0)} \dfrac{2xy}{\sqrt{x^2+y^2}}$

3. Calculate the indicated partial derivatives.

   (a) $u = x\sin(x+2y)$; u_x, u_{xy}, u_{xx}
   (b) $z = \tan(x^2y) + \cot(xy^2)$; $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$
   (c) $f = \arcsin(x^2y^{-3})$; f_x, f_{xx}, f_y
   (d) $z = \ln(\cos(x-2y))$; z_{yxx}

4. Find all first partial derivatives of the functions given below.

   (a) $f(x,y) = x^7y^9 + x^5y^{-5} + 100x$
   (b) $f(x,y) = \dfrac{x+y}{x-y}$
   (c) $f(x,y) = \ln(x^2y)$
   (d) $f(x,y) = x^y$
   (e) $f(x,y,z) = x^{xz}$
   (f) $f(x,y,z,t) = x^{-1}t^3z^6\sqrt{y}$

5. Confirm that Clairaut's theorem holds for the following functions. That is, calculate f_{xy} and f_{yx} and show that they are equal.

   (a) $f(x,y) = \exp(x^4y^2 + x^3y^4)$
   (b) $f(x,y) = \cos(xy^2 + x^3)$

6. Which of the following are solutions of Heat equation : $\dfrac{\partial u}{\partial x} - \dfrac{\partial^2 u}{\partial y^2} = 0$?

   (a) $u = x^3 + y^4/4$
   (b) $u = x^3 - y^4/4$
   (c) $u = x^2 + 3xy^2$
   (d) $u = \ln\sqrt{4x+2y}$
   (e) $u = e^{-2x}\cos y - e^{-y}\cos 2x$
   (f) $u = \exp(x+y)$

7. Bonus. Find f_{yy} of a function $f(x,y)$ such that $f_{xx} = x + 4y$ and $f_{xy} = 4x - y$.