Mathematics 2000: Assignment #5, Winter 2004

Due Feb 18

1. Find the radius of convergence and the interval of convergence of each power series. Don't forget to check the convergence of the endpoints seperately.

   (a) $\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{n-1}$

   (b) $\sum_{n=1}^{\infty} \sqrt{n}(x-2)^n$

   (c) $\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$

   (d) $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n3^n}$

   (e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n}}{(2n)!}$

   (f) $\sum_{n=1}^{\infty} \frac{n(x+4)^n}{2n^2+1}$

   (g) $\sum_{n=2}^{\infty} (-1)^n \frac{(2x+3)^n}{n(\ln n)^2}$

   (h) $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n^3 8^n}$

   (i) $\sum_{n=1}^{\infty} n^n x^n$

2. Find a power series representation for the following functions using formula $(1-x)^{-1} = 1 + x + x^2 + ... = \sum_{n=0}^{\infty} x^n$. Determine their interval of convergence.

   (a) $f(x) = \frac{2}{3-x}$

   (b) $f(x) = \frac{x^2}{2x+4}$

   (c) $f(x) = \frac{x}{16-x^4}$

3. Find the MacLaurin series for $f(x)$ and its radius of convergence given **known** MacLaurin series for $\sin x$, $\cos x$, e^x, $arctanx$ (see lecture notes or your book).

   (a) $f(x) = \begin{cases} x^{-1}\sin(x^3), & x \neq 0, \\ 0, & x = 0 \end{cases}$

   (b) $f(x) = x\cos(3x)$

   (c) $f(x) = e^x - \cos(x)$

   (d) $f(x) = e^{4x} + \arctan(x/2)$.

4. Use the **definition** to find the Taylor series (centered at c) for the functions (calculate just the first three or four nonzero terms):

   (a) $f(x) = e^{2x}, c = 1$

   (b) $f(x) = \cos x, c = \frac{\pi}{4}$

   (c) $f(x) = \cot x, c = \frac{\pi}{2}$

   (d) $f(x) = \ln x, c = 2$

   (e) $f(x) = x^{-3}, c = -2$

   (f) $f(x) = (1-x)^{-2}, c = 0$

5. **Bonus Problem.** Find the sum of the series.

   Hint: find related power series and the function which it represents.

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{3^n n!}$

   (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{4^n(2n-1)}$

   (c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2^{n-1/2}(2n-1)!}$

   (d) $\sum_{n=1}^{\infty} \frac{n2^n}{3^{n-1}}$.