Due Feb 11, 2004

Mathematics 2000: Assignment #4, Winter 2004

1. How many terms do you need to find the sum with indicated accuracy?

   a) $\sum \frac{(-1)^n}{n^5}$, $|error| < 0.001$,

   b) $\sum \frac{(-1)^n n}{5^n}$, $|error| < 0.0001$,

   c) $\sum \frac{(-1)^n}{3^n n!}$, $|error| < 0.00001$.

2. Determine if each of the following series is absolutely convergent, conditionally convergent, or divergent.

   (a) $\sum_{n=1}^{\infty} \frac{\cos(2n)}{2^n}$

   (b) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^4+n}$

   (c) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^3+n}$

   (d) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{arccot(-n)}$

   (e) $\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$

   (f) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n \ln n}$

   (g) $\sum_{n=1}^{\infty} \frac{n^n}{3^{3n}}$

   (h) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2 2^n}{n!}$

   (i) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

3. Determine whether the following series converge or diverge.

   (a) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n^5+1}}{n^{7/3}+n^{5/3}+n^{1/3}}$

   (b) $\sum_{n=1}^{\infty} \frac{2^n}{(n+3)!}$

   (c) $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

   (d) $\sum_{n=2}^{\infty} \left(\frac{n^3-1}{2n^3+1}\right)^n$

   (e) $\sum_{n=1}^{\infty} \frac{n^3-1}{2n^3+1}$

   (f) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

   (g) $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{\sqrt{n}}$

   (h) $\sum_{n=1}^{\infty} \frac{\sin(1/n)}{\sqrt{n}}$

   (i) $\sum_{n=1}^{\infty} \left(\sqrt[n^2]{3}-1\right)$

4. Bonus Problem. The Koch Snowflake.

   To construct the snowflake curve start with the equilateral triangle with sides of length 1. Divide each side into 3 equal parts, construct an equilateral triangle on the middle part and then delete the middle part. Repear this procedure for each side of the resulting polygon. The snowflake curve is a curve that results from repeating this process infinitely.

   Show that the snowflake curve is infinitly long but encluses only a finite area. Find the area.

   Hint: Find the series which represents the area and then find its sum.