Mathematics 2000: Assignment #4, Winter 2004- Answers

1. How many terms do you need to find the sum with indicated accuracy?

    a) $\sum_{n=1}^{infty} \dfrac{(-1)^n}{n^5}$, $|error| < 0.001$,   Answer: at least 4 terms.

    b) $\sum_{n=1}^{infty} \dfrac{(-1)^n n}{5^n}$, $|error| < 0.0001$,   Answer: at least 6 terms.

    c) $\sum_{n=0}^{infty} \dfrac{(-1)^n}{3^n n!}$, $|error| < 0.00001$.   Answer: at least 7 terms $(a_0 + ...a_6)$.

2. Determine if each of the following series is absolutely convergent, conditionally convergent, or divergent.

    (a) $\sum_{n=1}^{\infty} \dfrac{\cos(2n)}{2^n}$ Answer: absolutely convergent.

    (b) $\sum_{n=1}^{\infty} \dfrac{(-1)^n n^2}{n^4 + n}$ Answer: absolutely convergent.

    (c) $\sum_{n=1}^{\infty} \dfrac{(-1)^n n^2}{n^3 + n}$ Answer: conditionally convergent.

    (d) $\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{arccot(-n)}$ Answer: divergent.

    (e) $\sum_{n=1}^{\infty} \left(\dfrac{n^2 + 1}{2n^2 + 1} \right)^n$ Answer: absolutely convergent.

    (f) $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{n \ln n}$ Answer: conditionally convergent.

    (g) $\sum_{n=1}^{\infty} \dfrac{n^n}{3^{3n}}$ Answer: divergent.

    (h) $\sum_{n=1}^{\infty} \dfrac{(-1)^n n^2 2^n}{n!}$ Answer: absolutely convergent.

    (i) $\sum_{n=1}^{\infty} \dfrac{n!}{n^n}$ Answer: absolutely convergent.

3. Determine whether the following series converge or diverge.

    (a) $\sum_{n=1}^{\infty} (-1)^n \dfrac{\sqrt{n^5 + 1}}{n^{7/3} + n^{5/3} + n^{1/3}}$ Answer: divergent. Compare with $\sum n^{1/6}$.

    (b) $\sum_{n=1}^{\infty} \dfrac{2^n}{(n + 3)!}$ Answer: convergent. Ratio test.

(c) $\displaystyle\sum_{n=1}^{\infty} n^2 e^{-n^3}$ Answer: convergent. Integral test or Ratio Test.

(d) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{n^3-1}{2n^3+1}\right)^n$ Answer: convergent. Root test $l = 1/2$.

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3-1}{2n^3+1}$ Answer: divergent. Div. test.

(f) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$ Answer: convergent. Root test. $L = 1/e$.

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{\sqrt{n}}$ Answer: convergent. A.S T.

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(1/n)}{\sqrt{n}}$ Answer: convergent. Compare with $\sum n^{-3/2}$.

(i) $\displaystyle\sum_{n=1}^{\infty} \left(\sqrt[n^2]{3} - 1\right)$ Answer: convergent. Compare with $\sum n^{-2}$.

4. Bonus Problem. The Koch Snowflake.

To construct the snowflake curve start with the equilateral triangle with sides of length 1. Divide each side into 3 equal parts, construct an equilateral triangle on the middle part and then delete the middle part. Repear this procedure for each side of the resulting polygon. The snowflake curve is a curve that results from repeating this process infinitely.

Show that the snowflake curve is infinitly long but encluses only a finite area. Find the area.

Hint: Find the series which represents the area and then find its sum.