Mathematics 2000: Assignment #3, Winter 2004- Answers

Due Feb 4

A professor had a file with convergent series and another file with divergent series. Accidently the files were mixed up. Please, help the professor to sort things out.

1. Usint the integral test.

a) $\sum_{n=1}^{\infty} \frac{1}{n^4}$ Convergent.

b) $\sum_{n=1}^{\infty} \frac{1}{n^{1/4}}$ Divergent

c) $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ Divergent. Use substitution $u = x^3 + 1$ for integration

d) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ Divergent. Use substitution $u = \ln x$ for integration

e) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ Convergent. Use substitution $u = \ln x$ for integration

f) $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$ Convergent. Use integration by parts.

2 Using the comparison tests

a) $\sum_{n=1}^{\infty} \frac{n^{3/2}}{n^4+n+1}$ Convergent. Compare to $\sum \frac{1}{n^{5/2}}$.

b) $\sum_{n=1}^{\infty} \frac{1}{(n^2+n+2)^{1/4}}$ Divergent. Compare to $\sum \frac{1}{n^{1/2}}$.

c) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2-n+7}}{n^3+n^2-125n+678}$ Convergent. Compare to $\sum \frac{1}{n^2}$.

d) $\sum_{n=1}^{\infty} \frac{1+\sin^2 n}{2^n}$ Convergent. Compare to $\sum \frac{1}{2^n}$.

e) $\sum_{n=1}^{\infty} \tan(\frac{1}{n})$ Divergent. Compare to $\sum \frac{1}{n}$.

f) $\sum_{n=1}^{\infty} \cot(\frac{1}{n^2})$ Divergent. $\lim_{n\to\infty} \cot(\frac{1}{n^2}) = \infty$.

g) $\sum_{n=1}^{\infty} \frac{5^n}{3^n+4}$ Divergent. Compare to geometric series $\sum \frac{5^n}{3^n}$.

h) $\sum_{n=1}^{\infty} \frac{3^n}{4^n+4}$ Convergent. Compare to geometric series $\sum \frac{3^n}{4^n}$.

i) $\sum_{n=1}^{\infty} \frac{2^n + 3^{n/2} + 2^{2n}}{4^n}$ Divergent. Compare to geometric series $\sum 1^n$.

j) $\sum_{n=1}^{\infty} \frac{(n+1)!}{(n+2)!}$ Divergent. Compare to $\sum \frac{1}{n}$.

k) $\sum_{n=1}^{\infty} \frac{n!}{(n+2)!}$ Convergent. Compare to $\sum \frac{1}{n^2}$.

l) $\sum_{n=1}^{\infty} \frac{n}{n!}$ Convergent. Compare e.g. to geometric series $\sum \frac{1}{2^n}$.

3. Using the alternating series test.

a) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^3+4}$ Convergent.

b) $\sum_{n=2}^{\infty} (-1)^n \frac{n}{\ln n}$ Divergent. $\lim_{n\to\infty} (\frac{n}{\ln n}) = \infty$.

c) $\sum_{n=1}^{\infty} (-1)^n \tan(\frac{1}{n})$ Convergent.

d) $\sum_{n=1}^{\infty} (-1)^n \cot(\frac{1}{n})$ Divergent. $\lim_{n\to\infty} \cot(\frac{1}{n}) = \infty$.

e) $\sum_{n=3}^{\infty} (-1)^n \frac{1}{\ln n}$ Convergent.

f) $\sum_{n=1}^{\infty} (-1)^n \frac{n^n}{n!}$ Divergent. $\lim_{n\to\infty} (\frac{n^n}{n!}) = \infty$.

Bonus problem Are the following series convergent or divergent?

Hint: Use the Integral test to show that it is divergent.

a) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)(\ln|\ln n|)}$

b) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)(\ln|\ln n|)(\ln|\ln|\ln n||)(\ln|\ln|\ln|\ln n|||)}$

c) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)(\ln|\ln n|)(\ln|\ln|\ln n||)(\ln|\ln|\ln|\ln n|||)\ldots(\ln|\ln \ldots. \ln n|)}$