

1. Find the limit of the sequence.

a) $\lim_{n \rightarrow \infty} n \sin \frac{1}{2n}$ b) $\lim_{n \rightarrow \infty} 2n \sin \frac{3}{n}$

c) $\lim_{n \rightarrow \infty} \left(1 + \frac{2}{n}\right)^{3n}$ d) $\lim_{n \rightarrow \infty} \left(\frac{n}{n+5}\right)^{-n}$.

2. Find the sum of the geometric series

a) $\sum_{n=0}^{\infty} \left(\frac{1}{\pi}\right)^n$ b) $\sum_{n=1}^{\infty} \left(\frac{1}{\pi}\right)^n$ c) $\sum_{n=2}^{\infty} \left(\frac{1}{\pi}\right)^n$

d) $\sum_{n=0}^{\infty} (-0.4)^n + (0.4)^n$ e) $\sum_{n=0}^{\infty} \frac{1}{3^{n/3}}$ f) $\sum_{n=1}^{\infty} \left(\frac{5^n}{6^{n-1}}\right)$.

3. Express the repeating decimal as a geometric series and write its sum as the ratio of two integers.

(a) 0.234234234234...

(b) 1.2343434343434...

4. Find the sum by telescoping

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 3n + 2}$

(b) $\frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \frac{1}{7 \cdot 9} + \frac{1}{9 \cdot 11} + \cdots$

5. Explain why the following series are divergent. Find first 3 partial sum for each of the series.

a) $\sum_{n=3}^{\infty} \ln \left(\frac{n-1}{n}\right)$ b) $\sum_{n=1}^{\infty} \frac{n-1}{n+1}$ c) $\sum_{n=1}^{\infty} \frac{2}{3n}$

d) $\sum_{n=0}^{\infty} \left(\frac{5}{4}\right)^n$ e) $\sum_{n=1}^{\infty} \frac{2^n}{n^3}$ f) $\sum_{n=1}^{\infty} n \sin \frac{\pi}{n}$

- 6.
- Bonus problem**

Find the sum.

(a) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)(n+3)}$

(c) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)(n+3)\cdots(n+k)}$, where $k \geq 3$.