Name: ______________________

Due Jan 28 2004 Student No.: ______________________

ASSIGNMENT #2. ANSWERS. **Mathematics 2000** WINTER 2004

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1. Find the limit of the sequence.

a) $\lim_{n\to\infty} n\sin\frac{1}{2n} = \frac{1}{2}$ b) $\lim_{n\to\infty} 2n\sin\frac{3}{n} = 6$

c) $\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^{3n} = e^6$ d) $\lim_{n\to\infty}\left(\frac{n}{n+5}\right)^{-n} = e^5.$

2. Find the sum of the geometric series

a) $\sum_{n=0}^{\infty}\left(\frac{1}{\pi}\right)^n = \frac{\pi}{\pi-1}$ b) $\sum_{n=1}^{\infty}\left(\frac{1}{\pi}\right)^n = \frac{1}{\pi-1}$ c) $\sum_{n=2}^{\infty}\left(\frac{1}{\pi}\right)^n = \frac{1}{\pi(\pi-1)}$

d) $\sum_{n=0}^{\infty}(-0.4)^n + (0.4)^n = \frac{50}{21}$ e) $\sum_{n=0}^{\infty}\frac{1}{3^{n/3}} = \frac{3^{1/3}}{3^{1/3}-1}$ f) $\sum_{n=1}^{\infty}(\frac{5^n}{6^{n-1}}) = 30.$

3. Express the repeating decimal as a geometric series and write its sum as the ratio of two integers.

(a) $0.234234234234... = \frac{234}{999}$

(b) $1.2343434343434... = \frac{611}{495}$

4. Find the sum by telescoping

(a) $\sum_{n=1}^{\infty}\frac{1}{n^2+3n+2} = \frac{1}{2}$

(b) $\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\frac{1}{7\cdot9}+\frac{1}{9\cdot11}+\cdots = \frac{1}{6}$

5. Explain why the following series are divergent. Find first 3 partial sum for each of the series.

a) $\sum_{n=3}^{\infty}\ln\left(\frac{n-1}{n}\right)$ Partial sum $S_N = \ln 2 - \ln(N+2)$. Its limit is $-\infty$ as $N\to\infty$. Thus the series diverges. Find first 3 partial sum are: $S_1 = \ln 2 - \ln 3$, $S_2 = \ln 2 - \ln 4$, $S_3 = \ln 2 - \ln 5$

b) $\sum_{n=1}^{\infty}\frac{n-1}{n+1}$ General term $a_n \to 1 \neq 0$ as $n\to\infty$. Thus by Divergence Test the series diverges. Find first 3 partial sum are: $S_1 = 0$, $S_2 = 1/3$, $S_3 = 1/3 + 1/2 = 5/6$.

c) $\sum_{n=1}^{\infty}\frac{2}{3n}$ This is 2/3 of the Harmonic series, which is a p-series for $p=1$ and thus is divergent. Find first 3 partial sum are: $S_1 = 2/3$, $S_2 = 2/3 + 1/3 = 1$, $S_3 = 1 + 2/9 = 11/9$.

d) $\sum_{n=0}^{\infty}\left(\frac{5}{4}\right)^n$ This is geometric series with $r = 5/4 > 1$. $S_0 = 1$, $S_1 = 9/4$, $S_2 = 61/16$.

e) $\sum_{n=1}^{\infty}\frac{2^n}{n^3}$ The limit of a_n is infinity as $n\to\infty$. Thus by Divergence Test the series is divergent. $S_1 = 2$, $S_2 = 5/2$, $S_3 = 5/2 + 8/27$.

f) $\sum_{n=1}^{\infty} n\sin\frac{\pi}{n}$ The limit of a_n is π as $n\to\infty$. Thus by Divergence Test the series is divergent. $S_1 = 0$, $S_2 = 2$, $S_3 = 2 + 3\sin(\pi/3) = 2 + 3\sqrt{3}/2$.

6. **Bonus problem**

Find the sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)} = \frac{1}{4}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)(n+3)} = \frac{1}{18}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)(n+3)...(n+k)}$, where $k \geq 3$.