# Mathematics 2000: Assignment #1, Winter 2004

1. Find the first 5 terms of following sequences.

a) $a_n = \dfrac{1}{n+1}, \quad n = 0, 1, 2, ....$ b) $a_n = \dfrac{n!}{(n+1)!}, \quad n = 0, 1, 2, ...$

c) $a_{n+1} = a_n^2 + 1, \ a_0 = 0.$ d) $a_{n+1} = (a_n + 1)^2, \ a_0 = 0.$

2. Find a formula for the general term $a_n$ of the following sequences, assuming that the pattern of the first few terms continues.

a) $\left\{-1, \dfrac{1}{4}, \dfrac{-1}{9}, \dfrac{1}{16}, \dfrac{-1}{25}, \dfrac{1}{36}, \dfrac{-1}{49} \dots\right\}$ b) $\left\{1, \dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16}, \dfrac{1}{32}, \dfrac{1}{64} \dots\right\}$

3. Determine if the following sequences converge or diverge. Find the limit of convergent sequences.

a) $a_n = \dfrac{n!}{3^n}$ b) $b_n = \dfrac{n^3}{n!}$ c) $c_n = (-1)^n \dfrac{2^n}{n^2+1}$

d) $b_n = \dfrac{n^2+1}{7n^2-1}$ e) $c_n = \cot\left(\dfrac{2}{n}\right)$ f) $a_n = \sin\left(\dfrac{\sqrt{n}}{\sqrt{n+1}}\right)$

g) $c_n = \dfrac{(-3)^n}{(n+1)!}$ h) $a_n = \arctan(-2n)$ i) $b_n = \dfrac{\ln(2x)}{\ln(x^2)}$

4. Determine whether the following sequences are increasing, decreasing, or not monotonic. Which ones are bounded? Do they have a limit?

a) $a_n = \dfrac{2}{\cos(\pi n)}$ b) $b_n = \dfrac{n-1}{n+1}$ c) $c_n = \sin(\pi n)$

5. Find the limit of the sequence defined by

$$a_1 = 1, \quad a_{n+1} = \frac{1}{1+a_n}$$

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**Bonus Problem** Identify if the following statement is true or false. Explain why. Give an example.

1) If a sequence if bounded then it has a limit.
2) If a sequence has a limit then it is bounded.
3) If a sequence is monotonic then it is bounded.
4) If a sequence is monotonic then it is convergent.
5) If a sequence is both monotonic and bounded then it must have a limit.
6) If a sequence is convergent then it must be monotonic.
7) If a sequence $\{a_n\}$, $n \geq 1$ is convergent then the sequence $\{1/a_n\}$, $n \geq 1$ is divergent.