Mathematics 2000: Assignment #1, Winter 2004

Answers

1. Find the first 5 terms of following sequences.

a) $a_n = \dfrac{1}{n+1}, \quad n = 0, 1, 2, ...$ $\qquad 1, \dfrac{1}{2}, \dfrac{1}{3}, \dfrac{1}{4}, \dfrac{1}{5}, ...$

b) $a_n = \dfrac{n!}{(n+1)!}, \quad n = 0, 1, 2, ...$ $\qquad 1, \dfrac{1}{2}, \dfrac{1}{3}, \dfrac{1}{4}, \dfrac{1}{5}, ...$

c) $a_{n+1} = a_n^2 + 1, \ a_0 = 0.$ $\qquad 0, 1, 2, 5, 26, ...$

d) $a_{n+1} = (a_n + 1)^2, \ a_0 = 0.$ $\qquad 0, 1, 4, 25, 676, ...$

2. Find a formula for the general term a_n of the following sequences, assuming that the pattern of the first few terms continues.

a) $\left\{-1, \dfrac{1}{4}, \dfrac{-1}{9}, \dfrac{1}{16}, \dfrac{-1}{25}, \dfrac{1}{36}, \dfrac{-1}{49} \cdots\right\}$ $\qquad \dfrac{(-1)^n}{n^2}, \ n = 1, 2, 3, ...$

b) $\left\{1, \dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16}, \dfrac{1}{32}, \dfrac{1}{64} \cdots\right\}$ $\qquad \dfrac{1}{2^n}, \ n = 0, 1, 2, ...$

3. Determine if the following sequences converge or diverge. Find the limit of convergent sequences.

a) $a_n = \dfrac{n!}{3^n}$ **Diverges to ∞**

b) $b_n = \dfrac{n^3}{n!}$ **Converges to 0**.

c) $c_n = (-1)^n \dfrac{2^n}{n^2+1}$ **Diverges. Limit does not exist.**

d) $b_n = \dfrac{n^2+1}{7n^2-1}$ **Converges to 1/7**.

e) $c_n = \cot\left(\dfrac{2}{n}\right)$ **Diverges to ∞**.

f) $a_n = \sin\left(\dfrac{\sqrt{n}}{\sqrt{n+1}}\right)$ **Converges to** $\sin 1$.

g) $c_n = \dfrac{(-3)^n}{(n+1)!}$ **Converges to 0**.

h) $a_n = \arctan(-2n)$ **Converges to** $-\pi/2$.

i) $b_n = \dfrac{\ln(2x)}{\ln(x^2)}$ **Converges to 1/2**.

4. Determine whether the following sequences are increasing, decreasing, or not monotonic. Which ones are bounded? Do they have a limit?

a) $a_n = \dfrac{2}{\cos(\pi n)} = 2(-1)^n$. **It is bounded, non-monotonic and does not have a limit.**

b) $b_n = \dfrac{n-1}{n+1}$. **It is bounded, monotonic (increasing) and has limit 1.**

c) $c_n = \sin(\pi n) = 0$. **This is a constant sequence. Thus it is bounded and has limit** 0.

5. Find the limit of the sequence defined by

$$a_1 = 1, \quad a_{n+1} = \frac{1}{1 + a_n}$$

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limit$= (\sqrt{5} - 1)/2$

Bonus Problem Identify if each of the following statements is true or false. Explain why. Give an example.

1) If a sequence if bounded then it has a limit. **False**
2) If a sequence has a limit then it is bounded.**True**
3) If a sequence is monotonic then it is bounded.**False**
4) If a sequence is monotonic then it is convergent.**False**
5) If a sequence is both monotonic and bounded then it must have a limit.**True**
6) If a sequence is convergent then it must be monotonic.**False**
7) If a sequence $\{a_n\}$, $n \geq 1$ is convergent then the sequence $\{1/a_n\}$, $n \geq 1$ is divergent.**False**