MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SAMPLE FINAL EXAM **Mathematics 2000** WINTER 2006

Marks

1. Explain why the sequence is convergent and find the limit of the sequence.

[5] (a)

$$a_1 = 1/2, \quad a_{n+1} = \frac{2}{1+a_n}, \; n \geq 1$$

Answer 1.

[5] (b)

$$a_n = \frac{\cos(\pi n)}{1+n^3} - \sin(\pi n)$$

Answer 0.

2. Is the following series convergent or divergent? Explain. Identify the test used.

[5] (a)

$$\sum_{n=1}^{\infty} \frac{n^3 - n^2 + n + 5}{\sqrt{4n^6 + n - 1}}$$

Answer Divergent by Div test.

[5] (b)

$$\sum_{n=1}^{\infty} n \tan(1/n)$$

Answer Divergent by Div test.

[5] (c)

$$\sum_{n=2}^{\infty} \frac{1}{(\ln n)^2 n}$$

Answer Convergent by Integral test.

[5] (d)

$$\sum_{n=1}^{\infty} \left(1 - \frac{3}{n}\right)^{n^2}$$

Answer Convergent by Root test.

[8] 3. Is the following series conditionally or absolutely convergent? Identify the test used. Verify the conditions of the test.

$$\sum_{n=2}^{\infty} (-1)^n \sin(n^{-1/2}).$$

Answer Conditionally convergent, not absolutly.

4. Find the interval of convergence.

[7] (a)

$$\sum_{n=1}^{\infty} \frac{(x-100)^n}{3^n n^8 (2n)!}$$

Answer all real numbers.

[7] (b)

$$\sum_{n=1}^{\infty} \frac{(x+100)^n}{3^n n^8}$$

Answer $[-103, -97]$

[7] 5. Find the sum

$$\sum_{n=2}^{\infty} \frac{2^{n-2}}{\pi^n} - \frac{2}{(n+1)(n+2)}$$

Answer $(\pi^2 - 2\pi)^{-1} - 2/3$.

6. Find the coefficient a_{18} in the Maclaurin series $\sum_{n=0}^{\infty} a_n x^n$ for the function

[7] (a)

$$g(x) = x^6 \ln(2 + 3x^3)$$

Hint: Use Maclaurin formula for $\ln(1+x)$ and substitution. *Answer* $-3^4/(2^4 4)$

[5] (b)

$$f(x) = \frac{1}{(5 - x^6)^3}$$

Hint: Use binomial formula for $(1-x)^{-3}$.
Answer $\frac{2}{5^5}$.

[6] 7. Find the partial derivatives z_x, x_y and y_z for the function given implicitly

$$x^{100} \sin(xy^{10}) + y^2 \ln z^3 = xyz.$$

8. Consider function

$$F(x,y) = -x^3 + 4xy - 2y^2 + 1$$

[5] (a) Find critical points of the function $F(x,y)$.
Answer (0,0) and (4/3, 4/3).

[5] (b) Classify the critical points.
Answer saddle and local maxima.

9. Find volume of a solid bounded by paraboloid $z = -x^2 - y^2 + 4$ and planes $z = 0$ and $x = 0$. In other words, integrate function $f(x,y) = -x^2 - y^2 + 4$ over a half-circular domain $x^2 + y^2 \leq 4$ and $x \geq 0$.

   Repeat your calculations in both Cartesian and polar coordinates.

   Answer 4π.

[5] 10. (a) Evaluate changing the order of integration. Sketch the region of integration.

$$\int_0^1 \int_{2x}^2 \cos(1 + y^2)\, dydx$$

Answer $(\sin 2)/4$

[5] (b) Find $\cos(0.002)$ without a calculator with six correct digits. Explain.

[5] (c) Does the limit exist?

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{11x^2 + 17y^2}.$$

Answer no.