Math 2000: Assignment #7, Due Wed March 22, 2006

1. If the limit exists, find it. Otherwise show that it doesn't exist.

   (a) $\displaystyle\lim_{(x,y)\to(-3,4)} (x^3 + 3x^2y^2 - 2x + 3)$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(x^2 + y^2)}{x^2 + y^2}$

   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - y^6}{x^2 + y^6}$

   (d) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$

   (e) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^6y}{x^9 + y^3}$

   (f) $\displaystyle\lim_{(x,y)\to(0,0)} (1 + 2x^2y^2)^{\frac{1}{x^2y^2}}$

2. Calculate the indicated partial derivatives.

   (a) $u = xy\sec(xy)$; u_x, u_{xy}, u_{xx}

   (b) $z = \dfrac{x}{y} + \dfrac{y}{x}$; $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$

   (c) $xy + yz = zx$; x_y, y_z, z_x

   (d) $z = \ln(\sin(x - y))$; z_{yxx}

   (e) $f(x, y, z) = x^5 + x^4y^4z^3 + y^2x$; $\dfrac{\partial^3 f}{\partial x \partial y \partial z}$

   (f) $f(x, y, z) = xyz$; $f_x(0, 1, 2)$

3. Find all first partial derivatives of the functions given below.

   (a) $f(x, y) = x^3y^5 - 2x^3y^2 + 4x$

   (b) $f(x, y) = \dfrac{x - y}{x + y}$

   (c) $f(x, y) = \ln(x^2 + y^2)$

   (d) $f(x, t) = e^{\sin(t/x)}$

   (e) $f(x, y, z) = x^{yz}$

   (f) $f(x, y, z, t) = xy^3z^2\sqrt{t}$

4. Confirm that Clairaut's theorem holds for the following functions. That is, calculate f_{xy} and f_{yx} and show that they are equal.

   (a) $f(x, y) = x^4y^2 + x^3y^4 + xy^2 + x + y^2$

   (b) $f(x, y) = \sin(x^2y + x^2)$

5. Which of the following are solutions of Laplace's equation : $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$?

   (a) $u = x^2 + y^2$

   (b) $u = x^2 - y^2$

   (c) $u = x^3 + 3xy^2$

   (d) $u = \ln\sqrt{x^2 + y^2}$

   (e) $u = \sin x \cosh y + \cos x \sinh y$

   (f) $u = e^{-x}\cos y - e^{-y}\cos x$