Math 2000: Assignment #5, Due Feb 27

1. Find the radius of convergence and the interval of convergence of each power series. Don't forget to check the convergence of the endpoints seperately.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \sqrt{n} x^n$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{n(x-4)^n}{n^3+1}$

   (e) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{(2x+3)^n}{n \ln n}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdots (2n)}{1 \cdot 3 \cdot 5 \cdots (2n-1)} x^n$

   (g) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n^2 5^n}$

   (h) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (x-2)^n}{(2+n)!}$

2. Use the definition to find the Taylor series (centered at c) for the functions:

   (a) $f(x) = e^{3x}$, $c = 0$

   (b) $f(x) = \sin x$, $c = \frac{\pi}{4}$

   (c) $f(x) = \tan x$, $c = 0$ (calculate just the first three nonzero terms)

3. Find the MacLaurin series for $f(x)$ and its radius of convergence. You may use either the definition of a Maclaurin series or start with a known MacLaurin series for e^x, $(1+x)^k$, and $\tan^{-1} x$.

   (a) $f(x) = \arctan(x^2)$

   (b) $f(x) = xe^{2x}$

   (c) $f(x) = (1-3x)^{-5}$.

4. Find a power series representation for the following functions and determine their interval of convergence.

   (a) $f(x) = \dfrac{x}{4x+1}$

   (b) $f(x) = \dfrac{x^2}{a^3 - x^3}, (a \neq x)$.

5. Express $f(x) = \dfrac{7x-1}{3x^2+2x-1}$ as a power series by first using partial fractions. Find the interval of convergence.

6. Find a power series represetation for $f(x) = \ln(1+x)$. What is the radius of convergence? Use the above result to find a power series for $f(x) = x\ln(1+x)$.

7. If $f(x) = \dfrac{3}{x+2}$, find the power series for

   (a) $f(x)$ centered at 0,

   (b) $f(x)$ centered 1

   (c) $\dfrac{df}{dx}$ centered at 0

   (d) $\dfrac{df}{dx}$ centered at 1.

8. Find the sum of the power series:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n n x^{n-1}$.

9. Expand $f(x) = x^{-2}$ as a Taylor series around $c = 1$.