[7] 1. Use the limit definition of the definite integral to evaluate $\int_{-2}^{2} (x^2 + \pi x)\, dx$.

Answer

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n}((-2+4i/n)^2 + \pi(-2+4i/n)) = \frac{16}{3}$$

[3] 2. (a) If $y = \dfrac{\arctan 2x}{1+4x^2}$, find $\dfrac{dy}{dx}$.

Answer

$$\frac{2 - 8x\arctan(2x)}{(1+4x^2)^2}$$

[3] (b) Find $\dfrac{d}{dx}\left(\int_0^{3x^2} \dfrac{\arctan 2t}{1+4t^2}\, dt\right)$.

Answer

$$\frac{6x\arctan(6x^2)}{1+36x^4}$$

3. Find each of the following integrals:

[4] (a) $\displaystyle\int \frac{\sqrt[3]{5-\sqrt{x}}}{\sqrt{x}}\, dx$

Answer

$$\frac{-3(5-\sqrt{x})^{4/3}}{2} + C. \qquad \textbf{substitution } u = 5 - x^{1/2},$$

[4] (b) $\displaystyle\int \frac{\sin 2x}{\ln 2 + \cos 2x}\, dx$

Answer

$$\frac{-\ln|\ln 2 + \cos 2x|}{2} + C. \qquad \textbf{substitution } u = \ln 2 + \cos 2x$$

[4] (c) $\displaystyle\int \frac{2x+1}{\sqrt{9-x^2}}\, dx$

Answer

$$-2\sqrt{9-x^2} + \arcsin(x/3) + C. \qquad \textbf{split and substitution } u = 9 - x^2,$$

[4] (d) $\int x\sqrt{4-x}\,dx$

Answer

$$-\frac{8}{3}(4-x)^{3/2}+\frac{2}{5}(4-x)^{5/2}+C. \qquad \text{substitution } u=4-x$$

[4] (e) $\int \tan^3 2x \sec^3 2x\,dx$

Answer

$$\frac{1}{10}(\sec 2x)^5-\frac{1}{6}(\sec 2x)^3+C. \qquad \text{substitution } u=\sec x$$

[5] (f) $\int_0^2 \frac{x^2}{4+x^2}\,dx$

Answer

$$2-\frac{\pi}{2} \qquad \text{substitution } u=2\tan\Theta$$

4. Find each of the following integrals:

[5] (a)

$$\int \frac{\ln x^2}{x^2}\,dx = -\frac{2}{x}\ln x-\frac{2}{x}+C, \qquad \text{by parts}$$

[6] (b)

$$\int \frac{x}{\sqrt{x^2+4x+8}}\,dx = 2\sqrt{x^2+4x+8}-4\ln|\sqrt{x^2+4x+8}+x+2|+C$$

Complete the square. Then let $u=x+2$ and then $u=2\tan\Theta$.

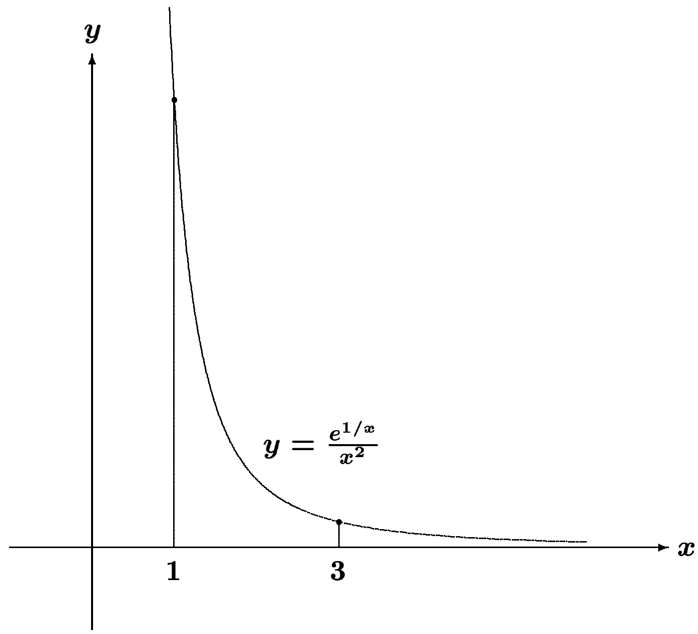
[6] (c) $\int \frac{x^2-2x-1}{(x-1)^2(x^2+1)}\,dx$ Partial fractions.

[5] (d)

$$\int \frac{\sqrt{x^2-4}}{x}\,dx = \sqrt{x^2-4}-2\operatorname{arcsec}(x/2)+C$$

Substution $x=2\sec\Theta$

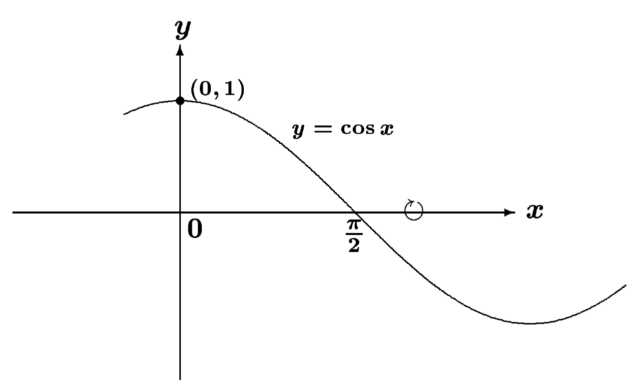
[4] 5. Find the area of the region bounded by the graphs of $y=\frac{e^{1/x}}{x^2}$ and $y=0$ from $x=1$ to $x=3$.



Answer $e - e^{1/3}$

6. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = \cos x,\ y = 0,\ x = 0,$ and $x = \frac{\pi}{2}$ about

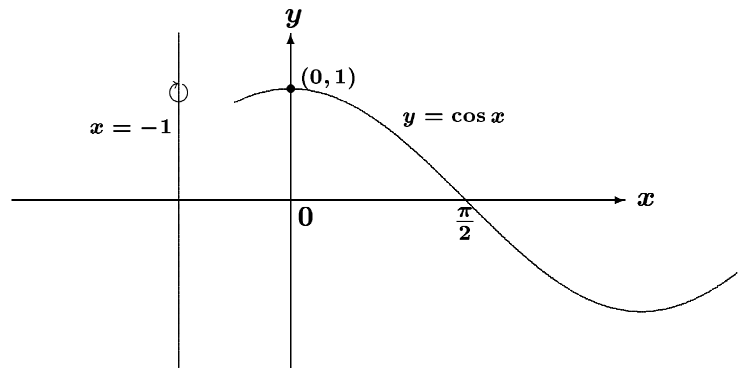
[6] (a) the x-axis.



Answer $\frac{\pi^2}{4}$

[6] (b) the line $x = -1$.

Answer π^2



[4] 7. From the following limits choose **the one** to which L' Hôpital's Rule may be applied and then evaluate this limit.

(a) $\displaystyle\lim_{x\to 0} \frac{4\cos^2 x}{x^2}$ (b) $\displaystyle\lim_{x\to\infty} \frac{\sin x^2}{x^2}$ (c) $\displaystyle\lim_{x\to 0} \frac{e^x - x - 1}{x^2}$.

Answer (c) $\displaystyle\lim_{x\to 0} \frac{e^x - x - 1}{x^2} = 1/2$.

8. Find each of the following:

[4] (a) $\displaystyle\lim_{x\to 0^+} \left(\frac{1}{x}\right)^x$

Answer 1.

[3] (b) $\displaystyle\int_2^3 \frac{2}{x\sqrt{x^2-4}}\,dx$

Answer arcsec (3/2).

[3] (c) $\displaystyle\int_0^\infty \frac{e^x}{1+e^{2x}}\,dx$

Answer $\lim_{a\to\infty} \arctan(e^a) - \arctan 1 = \pi/4$.

[10] 9. Do <u>**ANY TWO**</u> of the following:

(a) Use integration by parts to verify the reduction formula $\displaystyle\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx,$ where n is a positive integer, and then use the above formula to find $\displaystyle\int x^2 e^{3x}\,dx$.

Answer $u = x^n$, $dv = e^{ax}\,dx$, $\displaystyle\int x^2 e^{3x}\,dx = x^2e^{3x}/3 - (2/9)(xe^{3x} - e^{3x}/3) + C.$

(b) Use integration to obtain the formula for the area of a circle.

Answer

$$A = 4\int_0^R \sqrt{R^2 - x^2}\,dx = \pi R^2.$$

(c) Find the volume that remains after a hole of radius $\mathbf{3}$ is bored through the centre of a solid sphere of radius $\mathbf{5}$.

Answer

$$V = \pi \int_{-4}^{4} ((25 - x^2) - 9)\, dx = \frac{256\pi}{3}$$

(d) In a certain chemical reaction a substance S decomposes at a rate proportional to the amount of S present at time t. If an initial amount of this substance is reduced to 25 grams in 2 hours and 120 grams of the substance have decomposed in the first 4 hours, find the initial amount.

Answer 125 gramm

$$S(t) = S_0 e^{kt}, \quad S(2) = S_0 e^{2k} = 25, \quad S(4) = S_0 e^{4k} = S_0 - 120$$

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