[7]   1. Use the limit definition of the definite integral to evaluate $\displaystyle\int_{-2}^{2} (x^2 + \pi x)\, dx$.

[3]   2. (a) If $y = \dfrac{\arctan 2x}{1 + 4x^2}$, find $\dfrac{dy}{dx}$.

[3]   (b) Find $\dfrac{d}{dx}\left(\displaystyle\int_{0}^{3x^2} \dfrac{\arctan 2t}{1 + 4t^2}\, dt\right)$.

3. Find each of the following integrals:

[4]   (a) $\displaystyle\int \dfrac{\sqrt[3]{5 - \sqrt{x}}}{\sqrt{x}}\, dx$

[4]   (b) $\displaystyle\int \dfrac{\sin 2x}{\ln 2 + \cos 2x}\, dx$

[4]   (c) $\displaystyle\int \dfrac{2x + 1}{\sqrt{9 - x^2}}\, dx$

[4]   (d) $\displaystyle\int x\sqrt{4 - x}\, dx$

[4]   (e) $\displaystyle\int \tan^3 2x \sec^3 2x\, dx$

[5]   (f) $\displaystyle\int_{0}^{2} \dfrac{x^2}{4 + x^2}\, dx$

4. Find each of the following integrals:

[5]   (a) $\displaystyle\int \dfrac{\ln x^2}{x^2}\, dx$

[6]   (b) $\displaystyle\int \dfrac{x}{\sqrt{x^2 + 4x + 8}}\, dx$

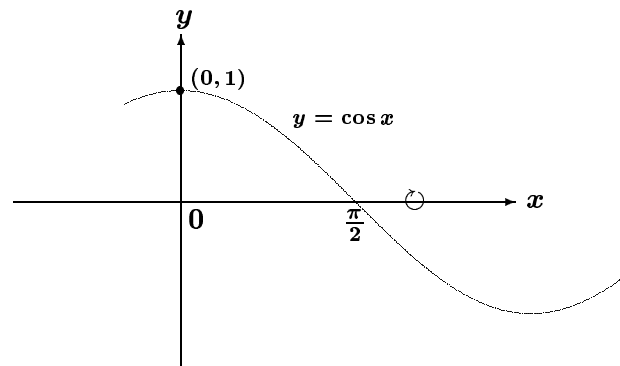[6]   (c) $\displaystyle\int \dfrac{x^2 - 2x - 1}{(x - 1)^2(x^2 + 1)}\, dx$

[5]   (d) $\displaystyle\int \dfrac{\sqrt{x^2 - 4}}{x}\, dx$

[4] 5. Find the area of the region bounded by the graphs of $y = \dfrac{e^{1/x}}{x^2}$ and $y = 0$ from $x = 1$ to $x = 3$.
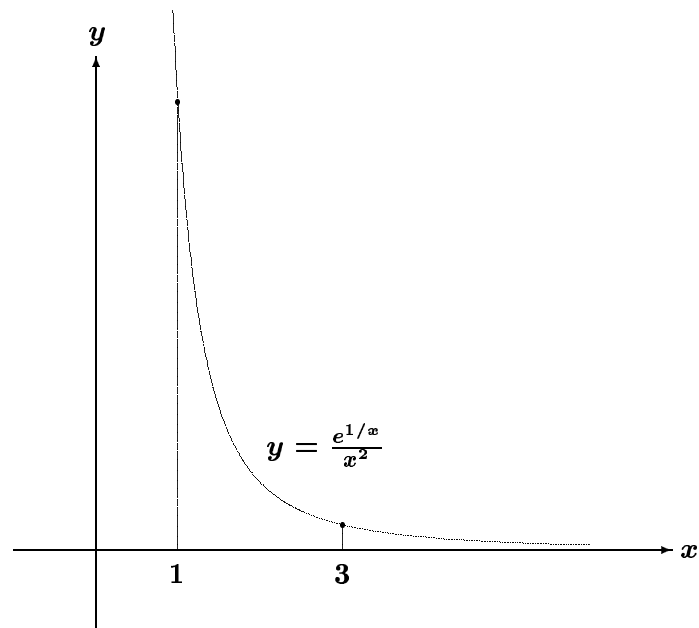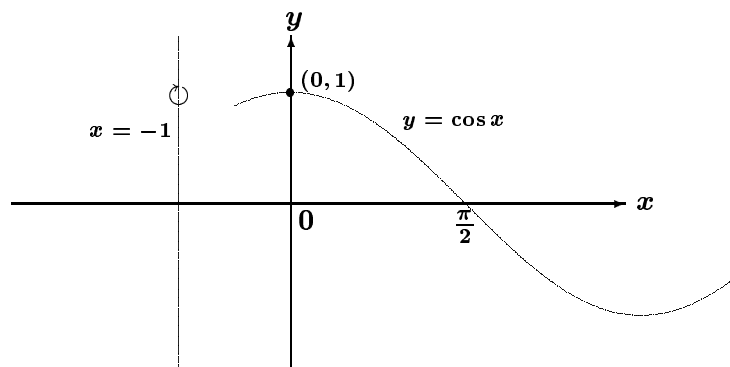

6. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = \cos x$, $y = 0$, $x = 0$, and $x = \frac{\pi}{2}$ about

[6] (a) the x-axis.

[6] (b) the line $x = -1$.

[4] 7. From the following limits choose **the one** to which L' Hôpital's Rule may be applied and then evaluate this limit.

(a) $\displaystyle\lim_{x \to 0} \frac{4\cos^2 x}{x^2}$ (b) $\displaystyle\lim_{x \to \infty} \frac{\sin x^2}{x^2}$ (c) $\displaystyle\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$.

8. Find each of the following:

[4] (a) $\displaystyle\lim_{x \to 0^+} \left(\frac{1}{x}\right)^x$

[3] (b) $\displaystyle\int_2^3 \frac{2}{x\sqrt{x^2 - 4}}\,dx$

[3] (c) $\displaystyle\int_0^\infty \frac{e^x}{1 + e^{2x}}\,dx$

[10] 9. Do **ANY TWO** of the following:

(a) Use integration by parts to verify the reduction formula
$$\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1} e^{ax}\,dx, \quad \text{where } n \text{ is a positive integer, and then}$$
use the above formula to find $\displaystyle\int x^2 e^{3x}\,dx$.

(b) Use integration to obtain the formula for the area of a circle.

(c) Find the volume that remains after a hole of radius **3** is bored through the centre of a solid sphere of radius **5**.

(d) In a certain chemical reaction a substance S decomposes at a rate proportional to the amount of S present at time t. If an initial amount of this substance is reduced to 25 grams in 2 hours and 120 grams of the substance have decomposed in the first 4 hours, find the initial amount.

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