Math 1001 **Solutions** **Assignment #8**

1. (a) $\int x\sec^2 x\,dx$ $\qquad u = x\,,\quad v' = \sec^2 x$
$\qquad u' = 1\,,\quad v = \tan x$

$$= x\tan x - \int \tan x\,dx = x\tan x - \ln|\sec x| + C$$

(b) $\int_2^4 \frac{\ln x}{x^2}\,dx$ $\qquad u = \ln x\,,\quad v' = \frac{1}{x^2}$
$\qquad u' = \frac{1}{x}\,,\quad v = -\frac{1}{x}$

$$= -\frac{1}{x}\ln x\,\Big|_2^4 + \int_2^4 \frac{1}{x^2}\,dx = \left(-\frac{1}{x}\ln x - \frac{1}{x}\right)\Big|_2^4 = \left(-\frac{1}{4}\ln 4 - \frac{1}{4}\right) - \left(-\frac{1}{2}\ln 2 - \frac{1}{2}\right)$$

$$= -\frac{1}{2}\ln 2 - \frac{1}{4} + \frac{1}{2}\ln 2 + \frac{1}{2} = \frac{1}{4}$$

(c) $\int \ln(x^2+4)\,dx$ $\qquad u = \ln(x^2+4)\,,\quad v' = 1$
$\qquad u' = \frac{2x}{x^2+4}\,,\quad v = x$

$$= x\ln(x^2+4) - \int \frac{2x^2}{x^2+4}\,dx = x\ln(x^2+4) - \int \left(2 - \frac{8}{x^2+4}\right)dx$$

$$= x\ln(x^2+4) - \left[2x - 8\cdot\frac{1}{2}\tan^{-1}\frac{x}{2}\right] + C = x\ln(x^2+4) - 2x + 4\tan^{-1}\frac{x}{2} + C$$

(d) $\int \tan^{-1}\frac{x}{4}\,dx$ $\qquad u = \tan^{-1}\frac{x}{4}\,,\quad v' = 1$
$\qquad u' = \frac{1}{1+\left(\frac{x}{4}\right)^2}\cdot\frac{1}{4} = \frac{4}{16+x^2}\,,\quad v = x$

$$= x\tan^{-1}\frac{x}{4} - \int \frac{4x}{16+x^2}\,dx = x\tan^{-1}\frac{x}{4} - 4\cdot\frac{1}{2}\ln|16+x^2| + C = x\tan^{-1}\frac{x}{4} - 2\ln(16+x^2) + C$$

(e) $\int \cos x\,\ln(\sin x)\,dx$ $\qquad u = \ln(\sin x)\,,\quad v' = \cos x$
$\qquad u' = \frac{1}{\sin x}\cdot\cos x\,,\quad v = \sin x$

$$= \sin x\,\ln(\sin x) - \int \cos x\,dx = \sin x\,\ln(\sin x) - \sin x + C$$

(f) $\int x^2\cos 3x dx$ $\qquad u = x^2\,,\quad v' = \cos 3x$
$\qquad u' = 2x\,,\quad v = \frac{1}{3}\sin 3x$

$$= \frac{x^2}{3}\sin 3x - \frac{2}{3}\int x\sin 3x\,dx \qquad u = x\,,\quad v' = \sin 3x$$

$$u' = 1\,,\quad v = -\frac{1}{3}\cos 3x$$

$$= \frac{x^2}{3}\sin 3x - \frac{2}{3}\left[-\frac{x}{3}\cos 3x + \frac{1}{3}\int \cos 3x\,dx\right] = \frac{x^2}{3}\sin 3x - \frac{2}{3}\left[-\frac{x}{3}\cos 3x + \frac{1}{9}\sin 3x\right] + C$$

$$= \frac{x^2}{3}\sin 3x + \frac{2x}{9}\cos 3x - \frac{2}{27}\sin 3x + C$$

(g) $\int x^2\tan^{-1}x\,dx \qquad u = \tan^{-1}x\,,\quad v' = x^2$

$$u' = \frac{1}{1+x^2}\,,\quad v = \frac{x^3}{3}$$

$$= \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int \frac{x^3}{1+x^2}\,dx \qquad \text{(divide out)}$$

$$= \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int \left(x - \frac{x}{x^2+1}\right)dx = \frac{x^3}{3}\tan^{-1}x - \frac{1}{6}x^2 + \frac{1}{6}\ln(x^2+1) + C$$

(h) $\int e^{2x}\sin 3x\,dx \qquad u = e^{2x}\,,\quad v' = \sin 3x$

$$u' = 2e^{2x}\,,\quad v = -\frac{1}{3}\cos 3x$$

$$= -\frac{1}{3}e^{2x}\cos 3x + \frac{2}{3}\int e^{2x}\cos 3x\,dx \qquad u = e^{2x}\,,\quad v' = \cos 3x$$

$$u' = 2e^{2x}\,,\quad v = \frac{1}{3}\sin 3x$$

$$= -\frac{1}{3}e^{2x}\cos 3x + \frac{2}{3}\left[\frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\int e^{2x}\sin 3x\,dx\right]$$

$$= -\frac{1}{3}e^{2x}\cos 3x + \frac{2}{9}e^{2x}\sin 3x - \frac{4}{9}\int e^{2x}\sin 3x\,dx$$

$$\frac{13}{9}\int e^{2x}\sin 3x\,dx = -\frac{1}{3}e^{2x}\cos 3x + \frac{2}{9}e^{2x}\sin 3x$$

$$\int e^{2x}\sin 3x\,dx = -\frac{3}{13}e^{2x}\cos 3x + \frac{2}{13}e^{2x}\sin 3x + C = \frac{e^{2x}}{13}\left(2\sin 3x - 3\cos 3x\right) + C$$

(i) $\int \sin(\ln x)\,dx \qquad u = \sin(\ln x)\,,\quad v' = 1$

$$u' = \cos(\ln x)\cdot\frac{1}{x}\,,\quad v = x$$

$$= x\sin(\ln x) - \int \cos(\ln x)\,dx \qquad u = \cos(\ln x) \quad , \quad v' = 1$$

$$u' = -\sin(\ln x)\cdot\frac{1}{x}, \quad v = x$$

$$= x\sin(\ln x) - \left[x\cos(\ln x) + \int \sin(\ln x)\,dx\right] = x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)\,dx$$

$$2\int \sin(\ln x)\,dx = x\sin(\ln x) - x\cos(\ln x)$$

$$\int \sin(\ln x)\,dx = \frac{x}{2}\,[\,\sin(\ln x) - \cos(\ln x)] + C$$

(j) $$\int \cos^3\frac{x}{3}\,dx = \int \cos^2\frac{x}{3}\cos\frac{x}{3}\,dx = \int\left(1-\sin^2\frac{x}{3}\right)\cos\frac{x}{3}\,dx \qquad u = \sin\frac{x}{3}$$

$$du = \frac{1}{3}\cos\frac{x}{3}\,dx$$

$$= 3\int(1-u^2)du = 3\left(u - \frac{u^3}{3}\right) + C = 3\sin\frac{x}{3} - \sin^3\frac{x}{3} + C$$

(k) $$\int \sin^5 x\,dx = \int \sin^4 x\cdot\sin x\,dx = \int(1-\cos^2 x)^2\sin x\,dx \qquad u = \cos x$$

$$du = -\sin x\,dx$$

$$= -\int(1-u^2)^2\,du = -\int(1-2u^2+u^4)\,du = -\left(u - \frac{2}{3}u^3 + \frac{1}{5}u^5\right) + C$$

$$= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$$

(l) $$\int \frac{\cos^3 x}{\sin^2 x}\,dx = \int \frac{\cos^2 x\cdot\cos x}{\sin^2 x}\,dx = \int \frac{(1-\sin^2 x)\cos x}{\sin^2 x}\,dx \qquad u = \sin x$$

$$du = \cos x\,dx$$

$$= \int \frac{1-u^2}{u^2}\,du = \int\left(\frac{1}{u^2} - 1\right)du = -\frac{1}{u} - u + C = -\frac{1}{\sin x} - \sin x + C = -\csc x - \sin x + C$$

(m) $$\int \sin^3 2x\,\cos^4 2x\,dx = \int \sin^2 2x\,\cos^4 2x\cdot\sin 2x\,dx = \int(1-\cos^2 2x)\cos^4 2x\cdot\sin 2x\,dx$$

$$u = \cos 2x$$

$$du = -2\sin 2x\,dx$$

$$= -\frac{1}{2}\int(1-u^2)u^4\,du = -\frac{1}{2}\int(u^4-u^6)\,du = -\frac{1}{2}\left(\frac{u^5}{5} - \frac{u^7}{7}\right) + C = -\frac{1}{10}\cos^5 2x + \frac{1}{14}\cos^7 2x + C$$

(n) $$\int \sin^4 2x\,dx = \int(\sin^2 2x)^2\,dx = \int\left(\frac{1-\cos 4x}{2}\right)^2 dx = \frac{1}{4}\int(1-2\cos 4x+\cos^2 4x)\,dx$$

$$= \frac{1}{4}\left[x - 2\cdot\frac{1}{4}\sin 4x + \int \cos^2 4x\,dx\right] = \frac{1}{4}\left[x - \frac{1}{2}\sin 4x + \int \frac{1+\cos 8x}{2}\,dx\right]$$

$$= \frac{1}{4}\left[x - \frac{1}{2}\sin 4x + \frac{1}{2}x + \frac{1}{16}\sin 8x\right] + C = \frac{3}{8}x - \frac{1}{8}\sin 4x + \frac{1}{64}\sin 8x + C$$

(o) $\displaystyle\int_0^{\frac{\pi}{6}} \tan^4 2x\, dx = \int_0^{\frac{\pi}{6}} \tan^2 2x \cdot \tan^2 2x\, dx = \int_0^{\frac{\pi}{6}} (\sec^2 2x - 1)\tan^2 2x\, dx$

$$= \int_0^{\frac{\pi}{6}} (\overbrace{\sec^2 2x \tan^2 2x}^{u=\tan 2x} - \tan^2 2x)\, dx = \frac{1}{2}\cdot \frac{\tan^3 2x}{3}\Big|_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} (\sec^2 2x - 1)\, dx$$

$$= \left(\frac{1}{6}\tan^3 2x - \frac{1}{2}\tan 2x + x\right)\Big|_0^{\frac{\pi}{6}} = \frac{(\sqrt{3}\,)^3}{6} - \frac{\sqrt{3}}{2} + \frac{\pi}{6} = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} + \frac{\pi}{6} = \frac{\pi}{6}$$

(p) $\displaystyle\int \sec^4 4\theta\, d\theta = \int \sec^2 4\theta\ \sec^2 4\theta\, d\theta = \int (1 + \tan^2 4\theta)\sec^2 4\theta\, d\theta$ $\qquad u = \tan 4\theta$, $du = 4\sec^2\theta\, d\theta$

$$= \frac{1}{4}\int (1 + u^2)\, du = \frac{1}{4}\left(u + \frac{u^3}{3}\right) + C = \frac{\tan 4\theta}{4} + \frac{\tan^3 4\theta}{12} + C$$

(q) $\displaystyle\int \tan^4 \frac{x}{2}\sec^4 \frac{x}{2}\, dx = \int \tan^4 \frac{x}{2}\sec^2 \frac{x}{2} \cdot \sec^2 \frac{x}{2}\, dx$

$$= \int \tan^4 \frac{x}{2}\left(1 + \tan^2 \frac{x}{2}\right)\sec^2 \frac{x}{2}\, dx \qquad u = \tan \frac{x}{2}, \quad du = \frac{1}{2}\sec^2 \frac{x}{2}$$

$$= 2\int u^4(1 + u^2)\, du = 2\int (u^4 + u^6)\, du = 2\left(\frac{u^5}{5} + \frac{u^7}{7}\right) + C = \frac{2}{5}\tan^5 \frac{x}{2} + \frac{2}{7}\tan^7 \frac{x}{2} + C$$

(r) $\displaystyle\int \tan^3 t \sec^5 t\, dt = \int \tan^2 t \sec^4 t \cdot \sec t \tan t\, dt = \int (\sec^2 t - 1)\sec^4 t \cdot \sec t \tan t\, dt$ $\qquad u = \sec t$, $du = \sec t \tan t dt$

$$= \int (u^2 - 1)u^4\, du = \int (u^6 - u^4)\, du = \frac{u^7}{7} - \frac{u^5}{5} + C = \frac{\sec^7 t}{7} - \frac{\sec^5 t}{5} + C$$

2. (a) $\displaystyle\int \frac{x}{\sqrt{3x+4}}\,dx$ $\qquad u = x\,, \quad v' = \dfrac{1}{\sqrt{3x+4}}$

$\qquad u' = 1\,, \quad v = \frac{2}{3}\sqrt{3x+4}$

$$= \frac{2}{3}\,x\sqrt{3x+4} - \frac{2}{3}\int \sqrt{3x+4}\,dx = \frac{2}{3}\,x\sqrt{3x+4} - \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{(3x+4)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{2}{3}\,x\sqrt{3x+4} - \frac{4}{27}(3x+4)^{\frac{3}{2}} + C$$

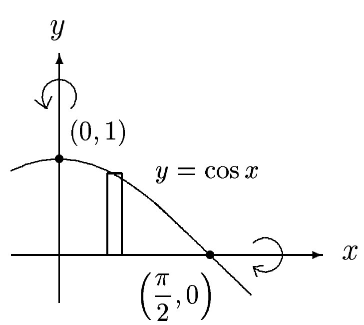
(b) $\displaystyle\int \frac{x}{\sqrt{3x+4}}\,dx$ $\qquad u = 3x+4 \quad \Rightarrow \quad x = \dfrac{u-4}{3}$

$\qquad du = 3dx$

$$= \int \frac{\frac{u-4}{3}}{\sqrt{u}}\cdot\frac{1}{3}\,du = \frac{1}{9}\int \frac{u-4}{\sqrt{u}}\,du = \frac{1}{9}\int \left(u^{\frac{1}{2}} - 4u^{-\frac{1}{2}}\right)du = \frac{1}{9}\left(\frac{u^{\frac{3}{2}}}{\frac{3}{2}} - 4\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right) + C$$

$$= \frac{2}{27}\,u^{3/2} - \frac{8}{9}\,u^{\frac{1}{2}} + C = \frac{2}{27}\,(3x+4)^{\frac{3}{2}} - \frac{8}{9}\,(3x+4)^{\frac{1}{2}} + C$$

3.



(a) $\displaystyle V = \int_0^{\frac{\pi}{2}} (\cos x)^2\,dx = \pi\int_0^{\frac{\pi}{2}} \cos^2 x\,dx = \pi\int\left(\frac{1+\cos 2x}{2}\right)dx$

$$= \pi\left(\frac{1}{2}\,x + \frac{1}{4}\,\sin 2x\right)\Bigg|_0^{\frac{\pi}{2}} = \pi\left[\left(\frac{\pi}{4} - 0\right) - (0-0)\right] = \frac{\pi^2}{4}$$

(b) $\displaystyle V = \int_0^{\frac{\pi}{2}} 2\pi x\cos x dx$ $\qquad u = x\,, \quad v' = \cos x$

$\qquad u' = 1\,, \quad v = \sin x$

$$= 2\pi x\sin x\Big|_0^{\frac{\pi}{2}} - 2\pi\int_0^{\frac{\pi}{2}} \sin x dx = 2\pi\left[\frac{\pi}{2}(1) - 0\right] + 2\pi\cos x\Big|_0^{\frac{\pi}{2}}$$

$$= \pi^2 + 2\pi(0-1) = \pi^2 - 2\pi = \pi(\pi - 2)$$

4. (a)

$$\int \cos^n x\,dx = \int \cos^{n-1} x \cos x\,dx \qquad \begin{array}{ll} u = \cos^{n-1} x &, \quad v' = \cos x \\ u' = -(n-1)\cos^{n-2} x \sin x\,, & v = \sin x \end{array}$$

$$= \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x \sin^2 x\,dx$$

$$= \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x\,(1-\cos^2 x)\,dx$$

$$= \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x - (n-1)\int \cos^n x\,dx$$

$$n\int \cos^n x\,dx = \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x\,dx$$

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

(b)

$$\int \cos^5 x\,dx = \frac{1}{5}\cos^4 x \sin x + \frac{4}{5}\int \cos^3 x\,dx = \frac{1}{5}\cos^4 x \sin x + \frac{4}{5}\left[\frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\int \cos x\,dx\right]$$

$$= \frac{1}{5}\cos^4 x \sin x + \frac{4}{15}\cos^2 x \sin x + \frac{8}{15}\sin x + C$$