1. (a) $\displaystyle\int \frac{x+3}{x^2+6x+7}\,dx$ $\qquad u = x^2+6x+7$, $\quad du = (2x+6)dx = 2(x+3)dx$

$$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+6x+7| + C$$

(b) $\displaystyle\int \frac{3t^4}{4-9t^5}\,dt$ $\qquad u = 4-9t^5$, $\quad du = -45t^4\,dt$

$$= 3\left(-\frac{1}{45}\right)\int \frac{1}{u}\,du = -\frac{1}{15}\ln|u| + C = -\frac{1}{15}\ln|4-9t^5| + C$$

(c) $\displaystyle\int_e^{e^2} \frac{4}{x\ln x}\,dx$ $\qquad u = \ln x$, $\quad du = \frac{1}{x}\,dx$ $\qquad x = e^2 \Rightarrow u = 2$, $\quad x = e \Rightarrow u = 1$

$$= 4\int_1^2 \frac{1}{u}\,du = 4\ln|u|\,\Big|_1^2 = 4\ln 2 - 4\ln 1 = 4\ln 2$$

(d) $\displaystyle\int \frac{1}{\sqrt{x}\,(1+4\sqrt{x}\,)}\,dx$ $\qquad u = 1+4\sqrt{x}$, $\quad du = \frac{2}{\sqrt{x}}\,dx$

$$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(1+4\sqrt{x}\,) + C$$

(e) $\displaystyle\int_0^{\frac{\pi}{6}} \frac{\sin 3x}{1+\cos 3x}\,dx$ $\qquad u = 1+\cos 3x$, $\quad du = -3\sin 3x\,dx$ $\qquad x = \frac{\pi}{6} \Rightarrow u = 1$, $\quad x = 0 \Rightarrow u = 2$

$$= -\frac{1}{3}\int_2^1 \frac{1}{u}\,du = -\frac{1}{3}\ln|u|\,\Big|_2^1 = -\frac{1}{3}\ln 1 + \frac{1}{3}\ln 2 = \frac{1}{3}\ln 2$$

(f)
$$\int \frac{1+\sin 3x}{\cos 3x}\,dx = \int\left(\frac{1}{\cos 3x} + \frac{\sin 3x}{\cos 3x}\right)dx = \int(\sec 3x + \tan 3x)dx$$
$$= \frac{1}{3}\ln|\sec 3x + \tan 3x| + \frac{1}{3}\ln|\sec 3x| + C$$

(g) $\displaystyle\int \frac{\ln x}{3x(1+\ln^2 x)}\,dx$ $\qquad u = 1+\ln^2 x$, $\quad du = \frac{2\ln x}{x}\,dx$

$$= \frac{1}{3}\cdot\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{6}\ln|u| + C = \frac{1}{6}\ln(1+\ln^2 x) + C$$

(h) $\displaystyle\int_{2\ln 2}^{3\ln 2} \frac{e^{2x}}{e^{2x}+8}\,dx$ $\quad u = e^{2x}+8$, $du = 2e^{2x}dx$ $\quad x = 3\ln 2 \Rightarrow u = 72$, $x = 2\ln 2 \Rightarrow u = 24$

$$= \frac{1}{2}\int_{24}^{72} \frac{1}{u}\,du = \frac{1}{2}\ln|u|\Big|_{24}^{72} = \frac{1}{2}(\ln 72 - \ln 24) = \frac{1}{2}\ln\frac{72}{24} = \frac{1}{2}\ln 3$$

(i) $\displaystyle\int \frac{\sec^2\theta}{1+5\tan\theta}\,d\theta$ $\quad u = 1+5\tan\theta$, $du = 5\sec^2\theta\,d\theta$

$$= \frac{1}{5}\int \frac{1}{u}\,du = \frac{1}{5}\ln|u| + C = \frac{1}{5}\ln|1+5\tan\theta| + C$$

(j) $\displaystyle\int \frac{\cot(\frac{4}{x})}{x^2}\,dx$ $\quad u = \frac{4}{x}$, $du = -\frac{4}{x^2}$

$$= -\frac{1}{4}\int \cot u\,du = -\frac{1}{4}\ln|\sin u| + C = -\frac{1}{4}\ln|\sin(\tfrac{4}{x})| + C$$

(k) $\displaystyle\int \frac{x^2}{(2x+1)^3}\,dx$ $\quad u = 2x+1 \Rightarrow x = \frac{u-1}{2}$, $du = 2dx$

$$= \frac{1}{2}\int \frac{\left(\frac{u-1}{2}\right)^2}{u^3}\,du = \frac{1}{8}\int \frac{u^2-2u+1}{u^3}\,du = \frac{1}{8}\int\left(\frac{1}{u} - \frac{2}{u^2} + \frac{1}{u^3}\right)du$$

$$= \frac{1}{8}\left(\ln|u| + \frac{2}{u} - \frac{1}{2u^2}\right) + C = \frac{1}{8}\ln|2x+1| + \frac{1}{4(2x+1)} - \frac{1}{16(2x+1)^2} + C$$

(l) $\displaystyle\int \frac{x^3-x^2+1}{x^3+1}\,dx$ $\quad$ Long division gives: $\displaystyle\frac{x^3-x^2+1}{x^3+1} = 1 - \frac{x^2}{x^3+1}$

$$= \int\left(1 - \frac{x^2}{x^3+1}\right)dx = x - \frac{1}{3}\ln|x^3+1| + C$$

(m) $\displaystyle\int \frac{2x+3}{2x-3}\,dx$ $\quad$ Long division gives: $\displaystyle\frac{2x+3}{2x-3} = 1 + \frac{6}{2x-3}$

$$= \int\left(1 + \frac{6}{2x-3}\right)dx = x + 6\cdot\frac{1}{2}\ln|2x-3| + C = x + 3\ln|2x-3| + C$$

2. (a) $\displaystyle\int x^3\sqrt{4x^2-1}\,dx$ $\quad u = 4x^2-1 \Rightarrow x^2 = \frac{u+1}{4}$, $du = 8x\,dx$

$$= \int x^2\sqrt{4x^2-1}\cdot x\,dx = \frac{1}{8}\int \frac{u+1}{4}\cdot\sqrt{u}\,du = \frac{1}{32}\int \left(u^{\frac{3}{2}}+u^{\frac{1}{2}}\right)du = \frac{1}{32}\left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}}+\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$

$$= \frac{1}{80}u^{\frac{5}{2}}+\frac{1}{48}u^{\frac{3}{2}}+C = \frac{1}{80}(4x^2-1)^{\frac{5}{2}}+\frac{1}{48}(4x^2-1)^{\frac{3}{2}}+C$$

(b) $\int \frac{x^5}{(x^2+4)^2}\,dx \qquad u = x^2+4 \quad\Rightarrow\quad x^2 = u-4$

$\qquad du = 2x\,dx$

$$= \int \frac{(x^2)^2}{(x^2+4)^2}\cdot x\,dx = \frac{1}{2}\int \frac{(u-4)^2}{u^2}\,du = \frac{1}{2}\int \frac{u^2-8u+16}{u^2}\,du = \frac{1}{2}\int\left(1-\frac{8}{u}+\frac{16}{u^2}\right)du$$

$$= \frac{1}{2}\left(u-8\ln|u|-\frac{16}{u}\right)+C = \frac{1}{2}(x^2+4)-4\ln(x^2+4)-\frac{8}{x^2+4}+C$$

3. Let $N(t)$ be the number of bacteria present at time t. Then $N'(t) = k\,N(t)$ and we have

$$\begin{aligned} N(t) &= N_0e^{kt} \\ &= 1500e^{kt} \end{aligned}$$

$N(5) = 4500 \quad\Rightarrow\quad 1500e^{5k} = 4500$

$\qquad e^{5k} = 3 \quad\Rightarrow\quad k = \frac{1}{5}\ln 3$

$N(t) = 1500e^{(\frac{1}{5}\ln 3)t}$

(i) $N(10) = 1500e^{(\frac{1}{5}\ln 3)10} = 1500e^{2\ln 3} = 1500(9) = 13,500$

(ii) $N(t) = 3000 \quad\Rightarrow\quad 1500e^{(\frac{1}{5}\ln 3)t} = 3000$

$\qquad e^{(\frac{1}{5}\ln 3)t} = 2$

$\qquad (\frac{1}{5}\ln 3)t = \ln 2 \quad\Rightarrow\quad t = \frac{5\ln 2}{\ln 3} \approx 3.15$ hours

4. Let $R(t)$ be the revenue at time t years after 2000.

$$\begin{aligned} R(t) &= R_0e^{kt} \\ &= 800,000e^{kt} \end{aligned}$$

$R(2) = 600,000 \quad\Rightarrow\quad 800,000e^{2k} = 600,000$

$\qquad e^{2k} = \frac{3}{4} \quad\Rightarrow\quad k = \frac{1}{2}\ln\frac{3}{4}$

$R(t) = 800,000e^{(\frac{1}{2}\ln\frac{3}{4})t}$

The expected revenue for 2003 would be

$$R(3) = 800,000e^{(\frac{1}{2}\ln\frac{3}{4})(3)} = 800,000e^{\frac{3}{2}\ln\frac{3}{4}} \approx \$519,615.24$$

5. Let $A(t)$ be the amount present at time t days.

$A(t) = A_0e^{kt}$

$$A(20) = \frac{1}{2} A_0 \quad \Rightarrow \quad A_0 e^{20k} = \tfrac{1}{2} A_0$$

$$e^{20k} = \tfrac{1}{2} \quad \Rightarrow \quad k = \tfrac{1}{20} \ln \tfrac{1}{2}$$

$$A(t) = A_0 e^{(\frac{1}{20} \ln \frac{1}{2})t}$$

(i) $A(10) = 50 \quad \Rightarrow \quad A_0 e^{(\frac{1}{20} \ln \frac{1}{2})(10)} = 50$

$$A_0 e^{\frac{1}{2} \ln \frac{1}{2}} = 50$$

$$A_0 = \frac{50}{e^{\frac{1}{2} \ln \frac{1}{2}}} = \frac{50}{(\frac{1}{2})^{\frac{1}{2}}} = 50\sqrt{2} \approx 70.7 \text{ mg}$$

(ii) $A(t) = \frac{1}{10} A_0 \quad \Rightarrow \quad A_0 e^{(\frac{1}{20} \ln \frac{1}{2})t} = \frac{1}{10} A_0$

$$\left(\frac{1}{20} \ln \frac{1}{2}\right) t = \ln \frac{1}{10}$$

$$t = \frac{20 \ln \frac{1}{10}}{\ln \frac{1}{2}} = \frac{20 \ln 10}{\ln 2} \approx 66.4 \text{ days}$$

6. Let $T(t)$ be the temperature of the object t hours after it is removed brom the furnace.

$$T(t) = T_m + (T_0 - T_m)e^{kt} \qquad T_m = 90\,,\ T_0 = 1500$$

$$T(t) = 90 + (1500 - 90)e^{kt}$$

$$= 90 + 1410e^{kt}$$

$T(1) = 1120 \quad \Rightarrow \quad 90 + 1410e^{k} = 1120$

$$1410e^{k} = 1030$$

$$e^{k} = \frac{103}{141}$$

$$k = \ln \frac{103}{141}$$

$$T(t) = 90 + 1410e^{(\ln \frac{103}{141})t}$$

So the temperature 5 hours after the unit is removed from the furnace is

$$T(5) = 90 + 1410e^{5 \ln \frac{103}{141}} \approx 383 \,\text{deg}$$