Math 1001 **Solutions** **Assignment #3**

1. (a) $\displaystyle\int \frac{x^3}{\sqrt{4-x^4}}\,dx$ $\qquad u = 4 - x^4$, $\quad du = -4x^3dx$

$$= -\frac{1}{4}\int \frac{1}{\sqrt{u}}\,du = -\frac{1}{4}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = -\frac{1}{2}\sqrt{u} + C = -\frac{1}{2}\sqrt{4-x^4} + C$$

(b) $\displaystyle\int_3^4 x\sqrt{25-x^2}\,dx$ $\qquad u = 25 - x^2$, $\quad du = -2xdx$ $\qquad x = 4 \Rightarrow u = 9$, $\quad x = 3 \Rightarrow u = 16$

$$= -\frac{1}{2}\int_{16}^{9}\sqrt{u}\,du = -\frac{1}{2}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_{16}^{9} = -\frac{1}{3}u^{\frac{3}{2}}\Bigg|_{16}^{9} = -\frac{1}{3}(27-64) = \frac{37}{3}$$

(c) $\displaystyle\int_0^1 \frac{2x^2}{(x^3+1)^4}\,dx$ $\qquad u = x^3 + 1$, $\quad du = 3x^2dx$ $\qquad x = 1 \Rightarrow u = 2$, $\quad x = 0 \Rightarrow u = 1$

$$= 2\cdot\frac{1}{3}\int_1^2 \frac{1}{u^4}\,du = \frac{2}{3}\cdot\frac{u^{-3}}{-3}\Bigg|_1^2 = -\frac{2}{9}\cdot\frac{1}{u^3}\Bigg|_1^2 = -\frac{2}{9}\left(\frac{1}{8}-1\right) = -\frac{2}{9}\left(-\frac{7}{8}\right) = \frac{7}{36}$$

(d) $\displaystyle\int \frac{e^{\frac{4}{x}}}{x^2}\,dx$ $\qquad u = \frac{4}{x}$, $\quad du = -\frac{4}{x^2}dx$

$$= -\frac{1}{4}\int e^u\,du = -\frac{1}{4}e^u + C = -\frac{1}{4}e^{\frac{4}{x}} + C$$

(e) $\displaystyle\int \frac{1}{x\ln^3 x}\,dx$ $\qquad u = \ln x$, $\quad du = \frac{1}{x}\,dx$

$$= \int \frac{1}{u^3}\,du = -\frac{1}{2u^2} + C = -\frac{1}{2\ln^2 x} + C$$

(f) $\displaystyle\int_1^9 \frac{1}{\sqrt{x}\,(1+\sqrt{x})^2}\,dx$ $\qquad u = 1 + \sqrt{x}$, $\quad du = \frac{1}{2\sqrt{x}}dx$ $\qquad x = 9 \Rightarrow u = 4$, $\quad x = 1 \Rightarrow u = 2$

$$= 2\int_2^4 \frac{1}{u^2}\,du = -\frac{2}{u}\Bigg|_2^4 = -\frac{1}{2} + 1 = \frac{1}{2}$$

(g) $\displaystyle\int_1^{e^2} \frac{(1+2\ln x)^3}{x}\,dx \qquad u = 1+2\ln x \qquad du = \frac{2}{x}\,dx \qquad x=e^2 \Rightarrow u=5 \qquad x=1 \Rightarrow u=1$

$$= \frac{1}{2}\int_1^5 u^3\,du = \frac{1}{2}\cdot\frac{u^4}{4}\Bigg|_1^5 = \frac{1}{8}(625-1) = \frac{1}{8}(624) = 78$$

(h) $\displaystyle\int \sqrt{x}\,(1+x\sqrt{x}\,)^5\,dx \qquad u = 1+x\sqrt{x} = 1+x^{\frac{3}{2}} \qquad dx = \tfrac{3}{2}\,x^{\frac{1}{2}}\,dx = \tfrac{3}{2}\,\sqrt{x}\,dx$

$$= \frac{2}{3}\int u^5\,du = \frac{2}{3}\cdot\frac{u^6}{6} + C = \frac{1}{9}\,u^6 + C = \frac{1}{9}\,(1+x\sqrt{x}\,)^6 + C$$

(i) $\displaystyle\int_0^{\frac{\pi}{12}} \frac{\sin 4x}{\cos^5 4x}\,dx \qquad u = \cos 4x \qquad du = -4\sin 4x\,dx \qquad x = \tfrac{\pi}{12} \Rightarrow u = \tfrac{1}{2} \qquad x = 0 \Rightarrow u = 1$

$$= -\frac{1}{4}\int_1^{\frac{1}{2}} \frac{1}{u^5}\,du = -\frac{1}{4}\cdot\frac{u^{-4}}{-4}\Bigg|_1^{\frac{1}{2}} = \frac{1}{16}\cdot\frac{1}{u^4}\Bigg|_1^{\frac{1}{2}} = \frac{1}{16}(16-1) = \frac{15}{16}$$

(j) $\displaystyle\int \sin^3 2x\;\cos 2x\,dx \qquad u = \sin 2x \qquad du = 2\cos 2x\,dx$

$$= \frac{1}{2}\int u^3\,du = \frac{1}{2}\cdot\frac{u^4}{4} + C = \frac{1}{8}\,\sin^4 2x + C$$

(k) $\displaystyle\int_0^{\frac{\pi}{3}} \sec^2 x\tan^4 x\,dx \qquad u = \tan x \qquad du = \sec^2 x\,dx \qquad x = \tfrac{\pi}{3} \Rightarrow u = \sqrt{3} \qquad x = 0 \Rightarrow u = 0$

$$= \int_0^{\sqrt{3}} u^4\,du = \frac{u^5}{5}\Bigg|_0^{\sqrt{3}} = \frac{1}{5}\,(9\sqrt{3}-0) = \frac{9\sqrt{3}}{5}$$

(l) $\displaystyle\int \frac{6\sin 2x}{(1-\cos 2x)^3}\,dx \qquad u = 1-\cos 2x \qquad du = 2\sin 2x\,dx$

$$= 6\cdot\frac{1}{3}\int\frac{1}{u^3}\,du = 3\cdot\frac{-1}{2u^2} + C = \frac{-3}{2u^2} + C = \frac{-3}{2(1-\cos 2x)^2} + C$$

(m) $\displaystyle\int \frac{e^{2x}}{9+4e^{2x}}\,dx$ $\qquad u = 9+4e^{2x}$, $\quad du = 8e^{2x}dx$

$$= \frac{1}{8}\int \frac{1}{u}\,du = \frac{1}{8}\ \ln|u| + C = \frac{1}{8}\ \ln|9+4e^{2x}| + C = \frac{1}{8}\ \ln(9+4e^{2x}) + C$$

(n) $\displaystyle\int_0^{\pi} \sin x\,\sqrt{5-4\cos x}\,dx$ $\qquad u = 5-4\cos x$, $\quad du = 4\sin x\,dx$ $\qquad x=\pi \Rightarrow u=9$, $\quad x=0 \Rightarrow u=1$

$$= \frac{1}{4}\int_1^9 \sqrt{u}\,du = \frac{1}{4}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_1^9 = \frac{1}{6}\,u^{\frac{3}{2}}\Big|_1^9 = \frac{1}{6}(27-1) = \frac{26}{6} = \frac{13}{3}$$

(o) $\displaystyle\int \frac{\sec^2(\frac{1}{x^2})}{x^3}\,dx$ $\qquad u = \frac{1}{x^2}$, $\quad du = -\frac{2}{x^3}\,dx$

$$= -\frac{1}{2}\int \sec^2 u\,du = -\frac{1}{2}\ \tan u + C = -\frac{1}{2}\ \tan(\tfrac{1}{x^2}) + C$$

(p) $\displaystyle\int \frac{2^x}{\sqrt{1+2^x}}\,dx$ $\qquad u = 1+2^x$, $\quad du = 2^x\ln 2\,dx$

$$= \frac{1}{\ln 2}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{\ln 2}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{2}{\ln 2}\,\sqrt{u} + C = \frac{2}{\ln 2}\,\sqrt{1+2^x} + C$$

2. $f'(x) = \dfrac{x-1}{\sqrt{x^2-2x+9}}$, $f(0) = 4$

$f(x) = \displaystyle\int \frac{x-1}{x^2-2x+9}\,dx$ $\qquad u = x^2-2x+9$, $\quad du = (2x-2)dx = 2(x-1)dx$

$$= \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\,\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = \sqrt{u} + C = \sqrt{x^2-2x+9} + C$$

$f(0) = 4 \ \Rightarrow\ 3 + C + 4 \ \Rightarrow\ c = 1.$ So

$$f(x) = \sqrt{x^2-2x+9} + 1$$

3. (a) $\displaystyle\int (4x+1)\sqrt{2x-1}\,dx \qquad u = 2x-1 \quad \Rightarrow \quad x = \frac{u+1}{2}$

$\qquad du = 2dx$

$$= \frac{1}{2}\int \left[4\left(\frac{u+1}{2}\right)+1\right]\sqrt{u}\,du = \frac{1}{2}\int (2u+3)\sqrt{u}\,du = \frac{1}{2}\int (2u^{\frac{3}{2}}+3u^{\frac{1}{2}})du$$

$$= \frac{1}{2}\left(2\cdot\frac{u^{\frac{5}{2}}}{\frac{5}{2}}+3\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right)+C = \frac{2}{5}\,u^{\frac{5}{2}}+u^{\frac{3}{2}}+C = \frac{2}{5}\,(2x-1)^{\frac{5}{2}}+(2x-1)^{\frac{3}{2}}+C$$

(b) $\displaystyle\int \frac{x^2}{\sqrt{x+3}}\,dx \qquad u = x+3 \quad \Rightarrow \quad x = u-3$

$\qquad du = dx$

$$= \int \frac{(u-3)^2}{\sqrt{u}}\,du = \int \frac{u^2-6u+9}{\sqrt{u}}\,du = \int (u^{\frac{3}{2}}-6u^{\frac{1}{2}}+9u^{-\frac{1}{2}})\,du = \frac{u^{\frac{5}{2}}}{\frac{5}{2}} - 6\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + 9\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \frac{2}{5}\,(x+3)^{\frac{5}{2}} - 4(x+3)^{\frac{3}{2}} + 18(x+3)^{\frac{1}{2}} + C$$

4. $y = \dfrac{x}{2x^2+1} \geq 0$ on $[1,2]$ and is continuous on $[1,2]$. So the area is

$$A = \int_1^2 \frac{x}{2x^2+1}\,dx \qquad u = 2x^2+1 \qquad x=2 \;\Rightarrow\; u=9$$

$$du = 4x\,dx \qquad x=1 \;\Rightarrow\; u=3$$

$$= \frac{1}{4}\int_3^9 \frac{1}{u}\,du = \frac{1}{4}\ln|u|\,\Big|_3^9 = \frac{1}{4}\,(\ln 9 - \ln 3) = \frac{1}{4}\,\ln 3$$