Math 1001 **Solutions** **Assignment #2**

1. For $n = 4$ and the interval $[-1, 2]$, $\triangle x = \frac{2+1}{4} = \frac{3}{4}$ and the regular partition is

$$[-1, -\tfrac{1}{4}],\ [-\tfrac{1}{4}, \tfrac{1}{2}],\ [\tfrac{1}{2}, \tfrac{5}{4}],\ [\tfrac{5}{4}, 2]$$

The right-hand endpoints are $c_1 = -\frac{1}{4},\ c_2 = \frac{1}{2},\ c_3 = \frac{5}{4},\ c_4 = 2$

$$\begin{aligned}
\textstyle\sum_{i=1}^{4} f(c_i)\triangle x &= f(c_1)\triangle x + f(c_2)\triangle x + f(c_3)\triangle x + f(c_4)\triangle x \\
&= f(-\tfrac{1}{4})\cdot\tfrac{3}{4} + f(\tfrac{1}{2})\cdot\tfrac{3}{4} + f(\tfrac{5}{4})\cdot\tfrac{3}{4} + f(2)\cdot\tfrac{5}{4} \\
&= \left[1 + \left(-\tfrac{1}{4}\right)^2\right]\cdot\tfrac{3}{4} + \left[1 + (\tfrac{1}{2})^2\right]\cdot\tfrac{3}{4} + \left[1 + \left(\tfrac{5}{4}\right)^2\right]\cdot\tfrac{3}{4} + [1 + (2)^2]\cdot\tfrac{3}{4} \\
&= \frac{17}{16}\cdot\frac{3}{4} + \frac{5}{4}\cdot\frac{3}{4} + \frac{41}{16}\cdot\frac{3}{4} + 5\cdot\frac{3}{4} = \frac{51}{64} + \frac{15}{16} + \frac{123}{64} + \frac{15}{4} \\
&= \frac{51 + 60 + 123 + 240}{64} = \frac{474}{64} = \frac{237}{32}
\end{aligned}$$

2. (a) $\displaystyle\int_0^2 2x^3 dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\triangle x \qquad \triangle x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n},\ x_i = a + i\triangle x = 0 + i\Big(\frac{2}{n}\Big) = \frac{2i}{n}$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(\frac{2i}{n}\right)\cdot\frac{2}{n} = \lim_{n\to\infty}\sum_{i=1}^{n} 2\left(\frac{2i}{n}\right)^3\cdot\frac{2}{n} = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{32i^3}{n^4} = \lim_{n\to\infty}\left[\frac{32}{n^4}\sum_{i=1}^{n} i^3\right]$$

$$= \lim_{n\to\infty}\left[\frac{32}{n^4}\cdot\frac{n^2(n+1)^2}{4}\right] = 8$$

(b) $\displaystyle\int_{-2}^{1}(2x - x^2)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\triangle x \qquad \triangle x = \frac{b-a}{n} = \frac{1-(-2)}{n} = \frac{3}{n},\ x_i = a + i\triangle x = -2 + \frac{3i}{n}$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(-2 + \frac{3i}{n}\right)\cdot\frac{3}{n} = \lim_{n\to\infty}\sum_{i=1}^{n}\left[2\left(-2 + \frac{3i}{n}\right) - \left(-2 + \frac{3i}{n}\right)^2\right]\cdot\frac{3}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(-4 + \frac{6i}{n} - 4 + \frac{12i}{n} - \frac{9i^2}{n^2}\right)\cdot\frac{3}{n} = \lim_{n\to\infty}\sum_{i=1}^{n}\left(-8 + \frac{18i}{n} - \frac{9i^2}{n^2}\right)\cdot\frac{3}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(-\frac{24}{n} + \frac{54i}{n^2} - \frac{27i^2}{n^3}\right) = \lim_{n\to\infty}\left[-24\sum_{i=1}^{n} 1 + \frac{54}{n^2}\sum_{i=1}^{n} i - \frac{27}{n^3}\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty}\left[-\frac{24}{n}\cdot n + \frac{54}{n^2}\cdot\frac{n(n+1)}{2} - \frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right] = -24 + 27 - 9 = -6$$

3. $\displaystyle\lim_{||\triangle||\to 0}\sum_{i=1}^{n}(\sec^2 2c_i)\,\triangle x_i = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\sec^2 2x\,dx = \tfrac{1}{2}\tan 2x\,\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \tfrac{1}{2}(-\sqrt{3} - \sqrt{3}) = -\sqrt{3}$

4. (a) $$\int_4^9 \frac{6x-5}{2\sqrt{x}}\,dx = \int_4^9 (3x^{\frac{1}{2}} - \tfrac{5}{2}x^{-\frac{1}{2}})\,dx = \left(3\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{5}{2}\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right)\Bigg|_4^9 = (2x^{\frac{3}{2}} - 5x^{\frac{1}{2}})\Big|_4^9$$
$$= (54-15)-(16-10) = 33$$

(b) $$\int_{-4}^{-1} \frac{(3x-2)^2}{x^2}\,dx = \int_{-4}^{-1} \frac{9x^2-12x+4}{x^2}\,dx = \int_{-4}^{-1}\left(9 - \frac{12}{x} + \frac{4}{x^2}\right)dx = \left(9x - 12\ln|x| - \frac{4}{x}\right)\Bigg|_{-4}^{-1}$$
$$= (-9-0+4) - (-36 - 12\ln 4 + 1) = 30 + 12\ln 4$$

(c) $$\int_1^2 \frac{6+x^4\sin\pi x}{x^4}\,dx = \int_1^2\left(\frac{6}{x^4} + \sin\pi x\right)dx = \left(-\frac{2}{x^3} - \frac{1}{\pi}\cos\pi x\right)\Bigg|_1^2 = \left(-\frac{1}{4} - \frac{1}{\pi}\right) - \left(-2 + \frac{1}{\pi}\right)$$
$$= \frac{7}{4} - \frac{2}{\pi} = \frac{7\pi - 8}{4\pi}$$

(d) $$\int_0^{\frac{1}{3}} \frac{4}{(2t-1)^3}\,dt = \int_0^{\frac{1}{3}} 4(2t-1)^{-3}\,dt = 4\cdot\frac{1}{2}\cdot\frac{(2t-1)^{-2}}{-2}\Bigg|_0^{\frac{1}{3}} = \frac{-1}{(2t-1)^2}\Bigg|_0^{\frac{1}{3}} = \frac{-1}{(-\frac{1}{3})^2} + \frac{1}{(-1)^2}$$
$$= -9 + 1 = -8$$

(e) $$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \csc\theta\,(\csc\theta + \cot\theta)\,d\theta = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\csc^2\theta + \csc\theta\cot\theta)\,d\theta = (-\cot\theta - \csc\theta)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} = (0-1) - (-\sqrt{3} - 2)$$
$$= 1 + \sqrt{3}$$

(f) $$\int_0^{\ln 2} \frac{e^{4x}+1}{e^{2x}}\,dx = \int_0^{\ln 2} (e^{2x} + e^{-2x})\,dx = (\tfrac{1}{2}e^{2x} - \tfrac{1}{2}e^{-2x})\Big|_0^{\ln 2} = [\tfrac{1}{2}(4) - \tfrac{1}{2}(\tfrac{1}{4})] - [\tfrac{1}{2}(1) - \tfrac{1}{2}(1)] = \frac{15}{8}$$

(g) $$\int_{-1}^0 \frac{3}{5x-4}\,dx = \frac{3}{5}\ln|5x-4|\Bigg|_{-1}^0 = \frac{3}{5}(\ln 4 - \ln 9) = \frac{3}{5}\ln\frac{4}{9}$$

(h) $$\int_0^3 |x^2-4|\,dx$$

<u>Note</u>: $|x^2-4| = -(x^2-4)$ for $x \in [0,2]$
$|x^2-4| = x^2-4$ for $x \in [2,3]$

$$= \int_0^2 |x^2-4|\,dx + \int_2^3 |x^2-4|\,dx = \int_0^2 -(x^2-4)\,dx + \int_2^3 (x^2-4)\,dx$$

$$= \left(-\frac{x^3}{3} + 4x\right)\Bigg|_0^2 + \left(\frac{x^3}{3} - 4x\right)\Bigg|_2^3 = \left(-\frac{8}{3} + 8\right) - (0+0) + (9-12) - \left(\frac{8}{3} - 8\right) = \frac{23}{3}$$

5. $F(x) = \int_4^{3x^2} \sqrt{2t+1}\; dt$

(a) $F'(x) = \sqrt{2(3x^2)+1} \cdot \frac{d}{dx}(3x^2) = 6x\sqrt{6x^2+1}$

(b) $\int_4^{3x^2} \sqrt{2t+1}\; dt = \frac{1}{2} \cdot \frac{(2t+1)^{\frac{3}{2}}}{\frac{3}{2}} \Big|_4^{3x^2} = \frac{1}{3}\,(2t+1)^{\frac{3}{2}} \Big|_4^{3x^2} = \frac{1}{3}\,(6x^2+1)^{\frac{3}{2}} - 9$

Then differentiating gives

$$F'(x) = \frac{1}{3} \cdot \frac{3}{2}\,(6x^2+1)^{\frac{1}{2}} \cdot 12x = 6x\sqrt{6x^2+1}$$

6. (a) $F(x) = \int_{\sqrt{x}}^{1} \frac{4t^3}{t^2+1}\, dt = -\int_1^{\sqrt{x}} \frac{4t^3}{t^2+1}\, dt$

$$F'(x) = -\frac{4(\sqrt{x}\,)^3}{(\sqrt{x}\,)^2+1} \cdot \frac{d}{dx}(\sqrt{x}\,) = -\frac{4x\sqrt{x}}{x+1} \cdot \frac{1}{2\sqrt{x}} = \frac{-2x}{x+1}$$

$$F'(\tfrac{1}{3}) = \frac{-\frac{2}{3}}{\frac{1}{3}+1} = \frac{-\frac{2}{3}}{\frac{4}{3}} = -\frac{1}{2}$$

(b) $F(x) = \int_{-x}^{2x} \ln(t^2+2)\, dt = \int_{-x}^{0} \ln(t^2+2)\, dt + \int_0^{2x} \ln(t^2+2)\, dt$

$$= -\int_0^{-x} \ln(t^2+2)\, dt + \int_0^{2x} \ln(t^2+2)\, dt$$

$$F'(x) = -\ln\left((-x)^2+2\right) \cdot (-1) + \ln\left((2x)^2+2\right) \cdot (2) = \ln(x^2+2) + 2\ln(4x^2+2)$$