Resonant Perturbation Theory of Decoherence, Relaxation and Evolution of Entanglement for Quantum Bits

Marco Merkli
Department of Mathematics, Memorial University, St. John's, Canada

Collaborators:

Gennady Berman
Theoretical Division, Los Alamos National Laboratory, Los Alamos, USA

Fausto Borgonovi
Dipartimento di Matematica, Università Cattolica, Brescia, Italy

Michael Sigal
Department of Mathematics, University of Toronto, Toronto, Canada

2010 CSQ, San Diego, April 27, 2010

Outline

- Open quantum systems:
  Superconducting qubits in thermal environment

- New method: **Resonance approach**

- New results:
  - Expression for dynamics valid **for all times**
  - **Clustering** of matrix elements: classification of decoherence times
  - Application to **non-integrable systems**:
    decoherence-, entanglement survival/death/revival times

- Resolves some problems of master equation approach:
  - incorrect results for times $t > (coupling)^{-2}$
  - incorrect final state due to $\begin{cases} O(coupling^2) \text{ corrections} \\ \text{long-lived metastable states} \end{cases}$

Open Quantum Systems

- Total system: "system S" + "reservoir R" + "interaction"
- S: **superconducting qubit**, atom, molecule, oscillator; *few degrees of freedom*
- R: collection of spins or **oscillators**; *many degrees of freedom*, in thermal equilibrium at temperature $T \geq 0$
- Total system: Hamiltonian $H = H_{\rm S}+H_{\rm R}+H_{\rm I}$, dynamics of **total density matrix** $\rho_{\rm SR}$

$$\rho_{\rm SR}(t) = \mathrm{e}^{-\mathrm{i}tH/\hbar}\rho_{\rm SR}(0)\,\mathrm{e}^{\mathrm{i}tH/\hbar}$$

- **Reduced density matrix**: $\rho(t) = \mathrm{Tr}_{\rm R}\ \rho_{\rm SR}(t)$ partial trace over R

- **Time-scales**:

$\tau_{\rm S}$	isolated S $(\leftrightarrow \omega_{\rm S} = (E-E')/\hbar)$
$\tau_{\rm relax}$	relaxation time of S $(\leftrightarrow H_{\rm I})$
$\tau_{\rm R} = \frac{\hbar}{k_{\rm B}T}$	thermal reservoir correlation time

Quantum Optical Master Equation

[Legget et al. ‘81, Palma et. al. ‘96, Gardiner-Zoller ‘04, Weiss ‘99]

• **Finite system coupled to bosonic reservoir**

$$H = H_{\mathrm{S}} + \sum_k \hbar\omega_k a_k^\dagger a_k + G\sum_k g_k(a_k^\dagger + a_k)$$

H_{S}, G: $N \times N$ matrices, g_k: coupling function; reduced evolution

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = -\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{Tr}_{\mathrm{R}}\big[H_{\mathrm{I}}(t), [H_{\mathrm{I}}(s), \rho_{\mathrm{RS}}(s)]\big]\mathrm{d}s$$

• **Born-Markov approximation**: system relaxation much slower than decay of reservoir correlations (memory effects weak) + **Rotating wave approximation**: syst. relax. much slower than free system dynamics

$\Rightarrow$ **Quantum Optical Regime:** $\max\{\tau_{\mathrm{R}}, \tau_{\mathrm{S}}\} \ll \tau_{\mathrm{relax}}$

$\longrightarrow$ Lindblad form of Master Equation: $\rho(t) = \mathrm{e}^{t\mathcal{L}}\rho(0)$, markovian

Quantum Brownian Motion Master Equation

Damped harmonic oscillator [Caldeira-Leggett ‘83, Haake-Reibold ‘84, Unruh-Zurek ‘89, Hu-Paz-Zhang ‘92]

$$H = \frac{p_0^2}{2m_0} + \frac{1}{2} m_0 \omega_0^2 q_0^2 + \sum_{n=1}^{N} \left[\frac{p_n^2}{2m_n} + \frac{1}{2} m_n \omega_n^2 q_n^2 \right] + q_0 \sum_{n=1}^{N} g_n q_n$$

Quadratic hamiltonian $\Rightarrow$ **exact** master equation (position representation):

$$\mathrm{i}\hbar \frac{\partial}{\partial t} \rho(x, x', t) = F\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial x'}, t \right) \rho(x, x', t)$$

F: complicated function encoding effects of reservoir

Quantum Brownian Motion Regime: $\tau_{\mathrm{R}} \ll \tau_{\mathrm{relax}}$ and $\tau_{\mathrm{R}} \ll \tau_{\mathrm{S}}$

$\longrightarrow$ Caldeira-Leggett master equation; $\rho(x, x, t)$ follows classical BM

Spectral density

- Effect of reservoir characterized by **spectral density**

$$J(\omega) = \sum_n \delta(\omega - \omega_n) \frac{g_n^2}{\omega_n}$$

- Limit of continuous modes of reservoir:

$$J(\omega) = \gamma \, \omega \left(\frac{\omega}{\Lambda}\right)^{n-1} \mathrm{e}^{-\omega^2/\Lambda^2}$$

$\gamma > 0$: measures overall size of coupling g_n
Λ: UV cutoff parameter (other forms of cutoff possible)

- $n = 1$ **ohmic**, $n > 1$ **superohmic**, $n < 1$ **subohmic** reservoir

Resonance Theory

N-level system coupled to reservoir(s)

$$H = H_{\mathrm{S}} + \sum_k \hbar\omega_k a_k^\dagger a_k + \lambda G \sum_k g_k (a_k^\dagger + a_k)$$

λ: coupling constant; free dynamics ($\lambda = 0$)

$$[\rho_t]_{mn} = \mathrm{e}^{\mathrm{i}t(E_n - E_m)/\hbar}[\rho_0]_{mn}$$

Effects of coupling to reservoirs:

- Irreversibility, energies become complex:

$$E_n - E_m \Rightarrow E_n - E_m + \lambda^2 \delta_{E_n - E_m} + O(\lambda^4)$$

  with $\mathrm{Im}\,\delta_{E_n - E_m} \geq 0$ (decay!)

- Clustering, $[\rho_t]_{mn}$ determined by $[\rho_0]_{kl}$ with $(k,l) \sim (m,n)$:

$$[\rho_t]_{mn} = F_t\Big([\rho_0]_{kl} \ : \ E_k - E_l = E_m - E_n\Big) + O(\lambda^2)$$

Method

• "**Complex scaling**, complex spectral deformation" à la Balslev-Combes '71 (Schrödinger operators), Feshbach resonance method

• H replaced by "**non-hermitian Hamiltonian**" K:

complex eigenvalues = resonance energies

eigenvectors = metastable states

• **Time-scales**:

$$\tau_{\rm S} = \max_{E\neq E'} \frac{\hbar}{E-E'}, \quad \tau_{\rm R} = \frac{\hbar}{k_{\rm B}T}, \quad \tau_{\rm relax} \propto \lambda^{-2}$$

• **Assumptions**:

– infra-red: $J(\omega)\sim\omega^n$ for $\omega\to 0$, $n=-1,1,3,5,\dots$

– ultra-violet: $J(\omega)\sim {\rm e}^{-\omega/\Lambda}, {\rm e}^{-\omega^2/\Lambda^2}$ (or similar) for $\omega\to\infty$

– λ small: $\max\{\tau_{\rm S},\tau_{\rm R}\}\ll\tau_{\rm relax}$

– $\Lambda^3 \ll \lambda^{-2}$

Result

For all times $t \geq 0$,

$$[\rho_t]_{mn} = \sum_{(k,l)\in\mathcal{C}(E_m-E_n)} A_t(m,n;k,l)[\rho_0]_{kl} \ + O(\lambda^2)$$

$O(\lambda^2)$ independent of t; different **matrix element clusters**

$$\mathcal{C}(E_m - E_n) := \{(k,l) \ : \ E_k - E_l = E_m - E_n\}$$

evolve independently; each cluster **markovian** evolution; Chapman-Kolmogorov relation

$$A_{t+s}(m,n;k,l) = \sum_{(p,q)\in\mathcal{C}(E_m-E_n)} A_t(m,n;p,q)A_s(p,q;k,l)$$

• Markov transition amplitudes A_t given by resonance data:

$$A_t(m,n;k,l) = \sum_{s=1}^{\mathrm{mult}(E_n-E_m)} \mathrm{e}^{\mathrm{i}t\varepsilon^{(s)}_{E_n-E_m}}\, C_{k,l;m,n}(s)$$

$C_{k,l;m,n}(s)$: overlap coefficient between resonance- and energy states of S

References:

Merkli-Sigal-Berman, *Phys. Rev. Lett.* (2007), *Annals of Physics* (2008)
Merkli-Berman-Sigal, *Annals of Physics* (2008) (decohernece)
Merkli-Berman-Borgonovi-Gebresellasie, *submitted* 2010 (entanglement)

Comparison: Master Equation and Resonance Approach

Advantages of RA

- **Extended time-range**

RA valid for $t \geq 0$, while ME resolves only times $t < \lambda^{-2}$:

– even for single qubit: ME predicts asymptotically Gibbs state $\propto \mathrm{e}^{-\beta H_{\mathrm{S}}}$, but true final state has corrections $O(\lambda^2)$ to Gibbs state

– H_{S} degenerate levels $\Rightarrow$ metastable states with lifetimes $\propto \lambda^{-n}$, $n > 2$; ME predicts **wrong stationary states**

- **Cluster Classification**

– different time-scales: each cluster has own decay $=$ decoherence time

– cluster containing diagonal relaxes to thermal values

– initially not populated clusters stay small $O(\lambda^2)$ forever

– for given quantum algorithm only a few clusters may be important $\Rightarrow$ only a few decoherence rates need analyzing

- **Applicability and Rigor**

RA applies to not exactly solvable systems, rigorous error control homogeneously in time, coincides with ME results where latter applicable

Limitations of RA

• RA (and MA) does not generally resolve variations of quantities of $O(\lambda^2)$

• RA assumes finite number N of degrees of freedom of S, due to condition $\tau_{\rm S} \ll \tau_{\rm relax}$, i.e., $\lambda^2 \ll \min(E-E') \sim 2^{-N}$

• Exact models show: for short times $t < \tau_\beta$, true dynamics can deviate significantly from markovian approximation ("initial slip"): both ME and RA may produce density matrices having negative eigenvalues (however RA correct up to $O(\lambda^2)$)

Possible extensions of RA

• *Non-markovian corrections*: matrix element clusters start to interact, time-homogeneous error reduced to $O(\lambda^4)$, or smaller

• *Overlapping resonances*: $\max(E-E'), \lambda^2 \ll \min\{1, k_{\rm B}T\}$

• *Time-dependent Hamiltonians*: e.g. $H_{\rm S}(t)$, $H_I(t)$ (slow variation and sudden jumps in two-level $H_{\rm S}$: [Merkli-Starr '09])

Resonance Theory: Decoherence

N-qubit register *collectively* coupled to single bosonic reservoir ($\hbar = 1$)

$$H_{\rm S} = \sum_{j=1}^N B_j S_j^z + \sum_{i,j=1}^N J_{ij} S_i^z S_j^z, \quad H_{\rm R} = \sum_k \omega_k a_k^\dagger a_k$$

B_j: magnetic field at the location of spin j, J_{ij}: pair interaction constants

- Interaction: collective energy conserving and energy exchange

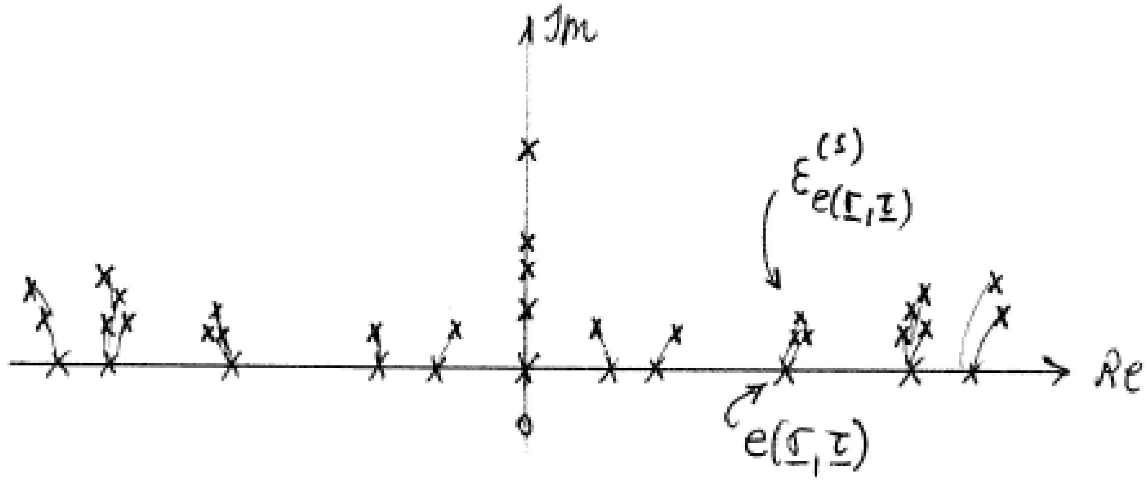
$$H_I = \lambda_1 \sum_{j=1}^N S_j^z \otimes \phi(g_1) + \lambda_2 \sum_{j=1}^N S_j^x \otimes \phi(g_2).$$

- $\phi(g_{1,2}) = \sum_k g_{1,2}(k)[a_k^\dagger + a_k]$
- Energy basis: $H_{\rm S}\varphi_{\underline{\sigma}} = E(\underline{\sigma})\varphi_{\underline{\sigma}}$, $E(\underline{\sigma}) = \sum_{j=1}^N B_j \sigma_j$
- Bohr energies: $e(\underline{\sigma}, \underline{\tau}) = E(\underline{\sigma}) - E(\underline{\tau})$
- Matrix element clusters: $\mathcal{C}(\underline{\sigma}, \underline{\tau}) = \{(\underline{\sigma}', \underline{\tau}') \ : \ e(\underline{\sigma}, \underline{\tau}) = e(\underline{\sigma}', \underline{\tau}')\}$

Resonance representation of dynamics

$$[\rho_t]_{\underline{\sigma},\underline{\tau}} \quad = \quad \sum_{(\underline{\sigma}',\underline{\tau}')\in\mathcal{C}(\underline{\sigma},\underline{\tau})} \ \sum_{s=1}^{\mathrm{mult}(e(\underline{\sigma},\underline{\tau}))} \exp\{\mathrm{i}t\varepsilon^{(s)}_{e(\underline{\sigma}',\underline{\tau}')}\}\ C(\underline{\sigma},\underline{\tau};\underline{\sigma}',\underline{\tau}')\ [\bar{\rho}_0]_{\underline{\sigma}',\underline{\tau}'}$$

$$+O(\lambda_1^2+\lambda_2^2)$$



- Perturbation expansion: $\varepsilon_e^{(s)} = e + \delta_e^{(s)} + O(\lambda_1^4+\lambda_2^4)$

Cluster decoherence rates

$$\gamma_e = \min\left\{\mathrm{Im}\varepsilon_e^{(s)}:\ s=1,\dots,\mathrm{mult}(e) \text{ s.t. } \varepsilon_e^{(s)}\neq 0\right\}$$

- Thermalization rate: $\gamma_{\mathrm{therm}} = \gamma_0$
- Assume *generic* magnetic fields: given any $n_j \in \{0,\pm1,\pm2\}$, the relation $\sum_{j=1}^N B_j n_j = 0$ implies $n_j = 0$ for all j (facilitates enumeration of register energies and eigenstates)
- **Results**

$$\gamma_e = \left\{\begin{array}{ll} \lambda_2^2 y_0, & e=0 \\ \lambda_1^2 y_1(e) + \lambda_2^2 y_2(e) + y_{12}(e), & e\neq 0 \end{array}\right\} + O(\lambda_1^4+\lambda_2^4)$$

y_1: due to energy conserving interaction; y_0, y_2: due to energy exchange interaction; y_{12}: due to both interactions, $O(\lambda_1^2+\lambda_2^2)$.

- $y_0 = 4\pi \min_{1\leq j\leq N}\{B_j^2\mathcal{G}_2(2B_j)\coth(\beta B_j)\}$ $\qquad(\mathcal{G}_2(x)\propto g_2(x))$
- $y_1(e) = \frac{\pi}{2\beta}\gamma_+ e_0^2(e)$ $\qquad(e_0(e) = \sum_{j=1}^N(\sigma_j-\tau_j)$, $\gamma_+ = \lim_{|k|\to 0}|k|g_1(k))$
- $y_2(e) = 2\pi\sum_{j:\sigma_j\neq\tau_j} B_j^2\mathcal{G}_2(2B_j)\coth(\beta B_j)$

- $y_{12}(e) \geq 0$: more complicated expression; > 0 unless λ_1 or λ_2 or $e_0(e)$ or γ_+ vanish; $y_{12}(e)$ approaches constant values as $T \to 0, \infty$

• **Full decoherence** $\gamma_e > 0$ **for all** $e \neq 0$: occurs for $\lambda_2 \neq 0$ and $g_2(2B_j) \neq 0$ for all j (provided λ_1, λ_2 small enough)

• **Dependence on register size** N

- Thermalization rate γ_0 independent of N
- Assume distribution of magnetic field $\langle\ \rangle$;

$$\langle y_1 \rangle = y_1 \propto e_0^2, \quad \langle y_2 \rangle \propto D(\underline{\sigma} - \underline{\tau}), \quad \langle y_{12} \rangle \propto N_0(e),$$

where $N_0(e) = \{\#j : \sigma_j = \tau_j\}$, $D(\underline{\sigma} - \underline{\tau}) := \sum_{j=1}^N |\sigma_j - \tau_j|$ is **Hamming distance** (N_0, D depend on e only)

- Decoherence rates:

∘ Pure energy-conserving interaction: $\gamma_e \propto \lambda_1^2 [\sum_{j=1}^N (\sigma_j - \tau_j)]^2$, can be as large as $O(\lambda_1^2 N^2)$

∘ Pure energy exchange interaction: $\gamma_e \propto \lambda_2^2 D(\underline{\sigma} - \underline{\tau}) \leq O(\lambda_2^2 N)$

∘ Both interactions: additional term $\langle y_{12}\rangle = O((\lambda_1^2+\lambda_2^2)N)$

Fastest decay rate of reduced off-diagonal density matrix elements:

– due to the energy conserving interaction alone $O(\lambda_1^2 N^2)$

– due to energy exchange interaction alone $O(\lambda_2^2 N)$

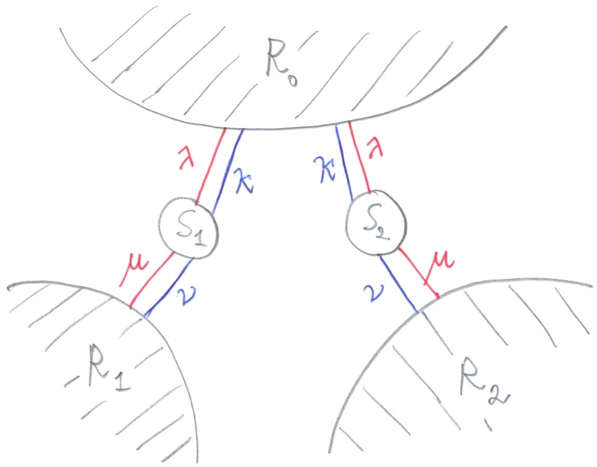
– relaxation of diagonal matrix elements $O(\lambda_1^2)$

Remarks:

- Local, energy-conserving interaction $\Rightarrow$ fastest decoherence rate $O(\lambda_1^2 N)$
- Assumption $\tau_{\rm S} \ll \tau_{\rm relax} \Leftrightarrow \lambda_{1,2}^2 \ll \Delta_N := \min^*_{\underline{\sigma},\underline{\tau}} |E(\underline{\sigma}) - E(\underline{\tau})|$
- Magnetic field roughly constant $B_j \sim B \Rightarrow \Delta_N \sim B$ indep. of N

Resonance Theory: Evolution of Entanglement

$$H = H_{\mathrm{S}_1} + H_{\mathrm{S}_2} + H_{\mathrm{R}_1} + H_{\mathrm{R}_2} + H_{\mathrm{R}_0} + W$$



$$W = \left.\begin{array}{l} \lambda(S_1^x + S_2^x) \otimes \varphi_0(g) \\ +\kappa(S_1^z + S_2^z) \otimes \varphi_0(f) \end{array}\right\} \text{collective}$$

$$\left.\begin{array}{l} +\mu\big(S_1^x \otimes \varphi_1(g) + S_2^x \otimes \varphi_2(g)\big) \\ +\nu\big(S_1^z \otimes \varphi_1(f) + S_2^z \otimes \varphi_2(f)\big) \end{array}\right\} \text{local}$$

energy exchange terms λ, μ, energy conserving terms κ, ν

$H_{\mathrm{S}_j} = B_j S_j^z$, $B_j > 0$ magnetic fields, S_j^z Pauli matrix, energies $\pm B_j$

$H_{\mathrm{R}_j} = \sum_k \omega_k \; a_{j,k}^\dagger a_{j,k}$, R_j at temperature $T = 1/\beta$

$\varphi_j(f) = \sum_k f_k a_{j,k}^\dagger + h.c.$

- Magnetic fields: $0 < B_1 < B_2$ s.t. $\frac{B_2}{B_1} \neq 2$ (avoids degeneracies)
- Transition energies: $\{0, \pm 2B_1, \pm 2B_2, \pm 2(B_2 - B_1), \pm 2(B_1 + B_2)\}$
- Matrix element clusters: $\mathcal{C}_1, \ldots, \mathcal{C}_5$

$$\begin{bmatrix} * & \bullet & \bullet & \diamond \\ & * & \diamond & \bullet \\ & & * & \bullet \\ & & & * \end{bmatrix}$$

(& hermitian)

$$\sigma_f(\omega) = \coth(\beta\omega/2) J_f(\omega), \;\; J_f(\omega) = \sum_k f_k^2 \delta(\omega - \omega_k) \text{ spectral density}$$

Coupling functions f = energy exchange, g = energy conserving

$$Y_2 = \tfrac{1}{2}\Big|\mathrm{Im}\,\Big[16\kappa_1^2\kappa_2^2 r^2 - (\lambda_2^2 + \mu_2^2)^2 \sigma_g^2(2B_2) - 8\mathrm{i}\kappa_1\kappa_2\,(\lambda_2^2 + \mu_2^2)\; r r_2'\Big]^{1/2}\Big|$$

$$Y_3 = \tfrac{1}{2}\Big|\mathrm{Im}\,\Big[16\kappa_1^2\kappa_2^2 r^2 - (\lambda_1^2 + \mu_1^2)^2 \sigma_g^2(2B_1) - 8\mathrm{i}\kappa_1\kappa_2\,(\lambda_1^2 + \mu_1^2)\; r r_1'\Big]^{1/2}\Big|$$

where $$r = \mathrm{P.V.} \int_{\mathbf{R}^3} \frac{|f|^2}{|k|} \mathrm{d}^3 k, \qquad r_j' = 4\pi B_j^2 \int_{S^2} |g(2B_j, \Sigma)|^2 \mathrm{d}\Sigma$$

Cluster decoherence rates

$2B_1$, $2B_2$: qubit transition energies

$$
\begin{aligned}
\gamma_{\rm therm} &= \min_{j=1,2}\left\{(\lambda_j^2+\mu_j^2)\sigma_g(2B_j)\right\} + O(\alpha^4)\\
\gamma_2 &= \tfrac12(\lambda_1^2+\mu_1^2)\sigma_g(2B_1) + \tfrac12(\lambda_2^2+\mu_2^2)\sigma_g(2B_2)\\
&\qquad -Y_2 + (\kappa_1^2+\nu_1^2)\sigma_f(0) + O(\alpha^4)\\
\gamma_3 &= \tfrac12(\lambda_1^2+\mu_1^2)\sigma_g(2B_1) + \tfrac12(\lambda_2^2+\mu_2^2)\sigma_g(2B_2)\\
&\qquad -Y_3 + (\kappa_2^2+\nu_2^2)\sigma_f(0) + O(\alpha^4)\\
\gamma_4 &= (\lambda_1^2+\mu_1^2)\sigma_g(2B_1) + (\lambda_2^2+\mu_2^2)\sigma_g(2B_2)\\
&\qquad +\left[(\kappa_1-\kappa_2)^2+\nu_1^2+\nu_2^2\right]\sigma_f(0) + O(\alpha^4)\\
\gamma_5 &= (\lambda_1^2+\mu_1^2)\sigma_g(2B_1) + (\lambda_2^2+\mu_2^2)\sigma_g(2B_2)\\
&\qquad +\left[(\kappa_1+\kappa_2)^2+\nu_1^2+\nu_2^2\right]\sigma_f(0) + O(\alpha^4)
\end{aligned}
$$

Discussion: decoherence rates

• Thermalization rate depends on energy-exchange coupling only.

• Purely energy-exchange interactions: $\kappa_j = \nu_j = 0 \Rightarrow$ rates depend symmetrically on local and collective influence through $\lambda_j^2 + \mu_j^2$.

• Purely energy-conserving interactions: $\lambda_j = \mu_j = 0 \Rightarrow$ rates do not depend symmetrically on local and collective terms. E.g. γ_4 may depend on local interaction only $(\kappa_1 = \kappa_2)$.

• Y_1 and Y_2 contain products of exchange and conserving terms.

Entanglement evolution

- Entanglement of formation [Bennet et al '96] of two qubits $\leftrightarrow$ concurrence [Wootters '97]:

$$C(\rho) = \max\left\{0, D(\rho)\right\}, \qquad D(\rho) = \sqrt{\nu_1} - \left[\sqrt{\nu_2} - \sqrt{\nu_3} - \sqrt{\nu_4}\right]$$

$\nu_1 \geq \nu_2 \geq \nu_3 \geq \nu_4 \geq 0$ eigenvalues of matrix $\xi := \rho(\sigma^y \otimes \sigma^y)\overline{\rho}(\sigma^y \otimes \sigma^y)$

- Dominant dynamics: only initially populated clusters have nontrivial dynamics
- Example: pure initial state $\psi_0 = a|++\rangle + b|--\rangle$

$$\rho_0 = \begin{bmatrix} p & 0 & 0 & u \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \overline{u} & 0 & 0 & 1-p \end{bmatrix} \Rightarrow \rho_t = \begin{bmatrix} x_1(t) & 0 & 0 & u(t) \\ 0 & x_2(t) & 0 & 0 \\ 0 & 0 & x_3(t) & 0 \\ \overline{u}(t) & 0 & 0 & x_4(t) \end{bmatrix} + O(\alpha^2)$$

- Initial concurrence: $C(\rho_0) = 2\sqrt{p(1-p)}$
- Dynamics

$$
\begin{aligned}
x_1(t) &= pA_t(11;11) + (1-p)A_t(11;44) \\
x_2(t) &= pA_t(22;11) + (1-p)A_t(22;44) \\
&\vdots \\
u(t) &= \mathrm{e}^{\mathrm{i}t\varepsilon_{2(B_1+B_2)}} u(0)
\end{aligned}
$$

$A_t(kk;ll) \leftarrow$ resonance energies bifurcating out of $e = 0$. Leading terms:

$$
\delta_2 = (\lambda_1^2 + \mu_1^2)\sigma_g(B_1), \quad \delta_3 = (\lambda_2^2 + \mu_2^2)\sigma_g(B_2), \quad \delta_4 = \delta_2 + \delta_3
$$

Leading term of $\mathrm{Im}\,\varepsilon_{2(B_1+B_2)}$:

$$
\delta_5 = \delta_2 + \delta_3 + [(\kappa_1 + \kappa_2)^2 + \nu_1^2 + \nu_2^2]\sigma_f(0)
$$

Entanglement death/survival times

Take coupling s.t. $\delta_2, \delta_3 > 0$ *(thermalization). There is a positive constant* α_0 *(independent of* p*) s.t. if* $0 < \alpha \le \alpha_0 \sqrt{p(1-p)}$*, then we have the following.*

Entanglement death time. *There is a constant* $C_A > 0$ *(independent of* p*,* α*) such that concurrence* $C(\rho_t) = 0$ *for all* $t \ge t_A$*, where*

$$t_A := \max\left\{ \frac{1}{\delta_5} \ln \left[C_A \frac{\sqrt{p(1-p)}}{\alpha^2} \right], \frac{1}{\delta_2 + \delta_3} \ln \left[C_A \frac{p(1-p)}{\alpha^2} \right] \right\}.$$

Entanglement survival time. *There is a constant* $C_B > 0$ *(independent of* p*,* α*) such that concurrence* $C(\rho_t) > 0$ *for all* $t \le t_B$*, where*

$$t_B := \frac{1}{\max\{\delta_2, \delta_3\}} \ln \left[1 + C_B \alpha^2 \right].$$

Discussion: entanglement evolution

• Result gives disentanglement bounds for the true dynamics of the qubits for non-integrable interactions

• Disentanglement time is *finite* since $\delta_2, \delta_3 > 0$ (which implies thermalization). If system does not thermalize then it may happen that entanglement stays nonzero for all times (it may decay or even stay constant)

• Rates δ_j are of order α^2. Both t_A and t_B increase with decreasing coupling strength

Entanglement creation

Braun '02: energy conserving collective coupling, initial product state $\frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) \otimes \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) \Rightarrow$ concurrence creation, death and revival

Dynamics in resonance approximation:

- *Purely energy-exchange coupling*

$[\rho_t]_{mn}$ depends on $\lambda^2 + \mu^2$ only $\Rightarrow$ *Creation of entanglement under purely collective and purely local energy-exchange dynamics is* the same

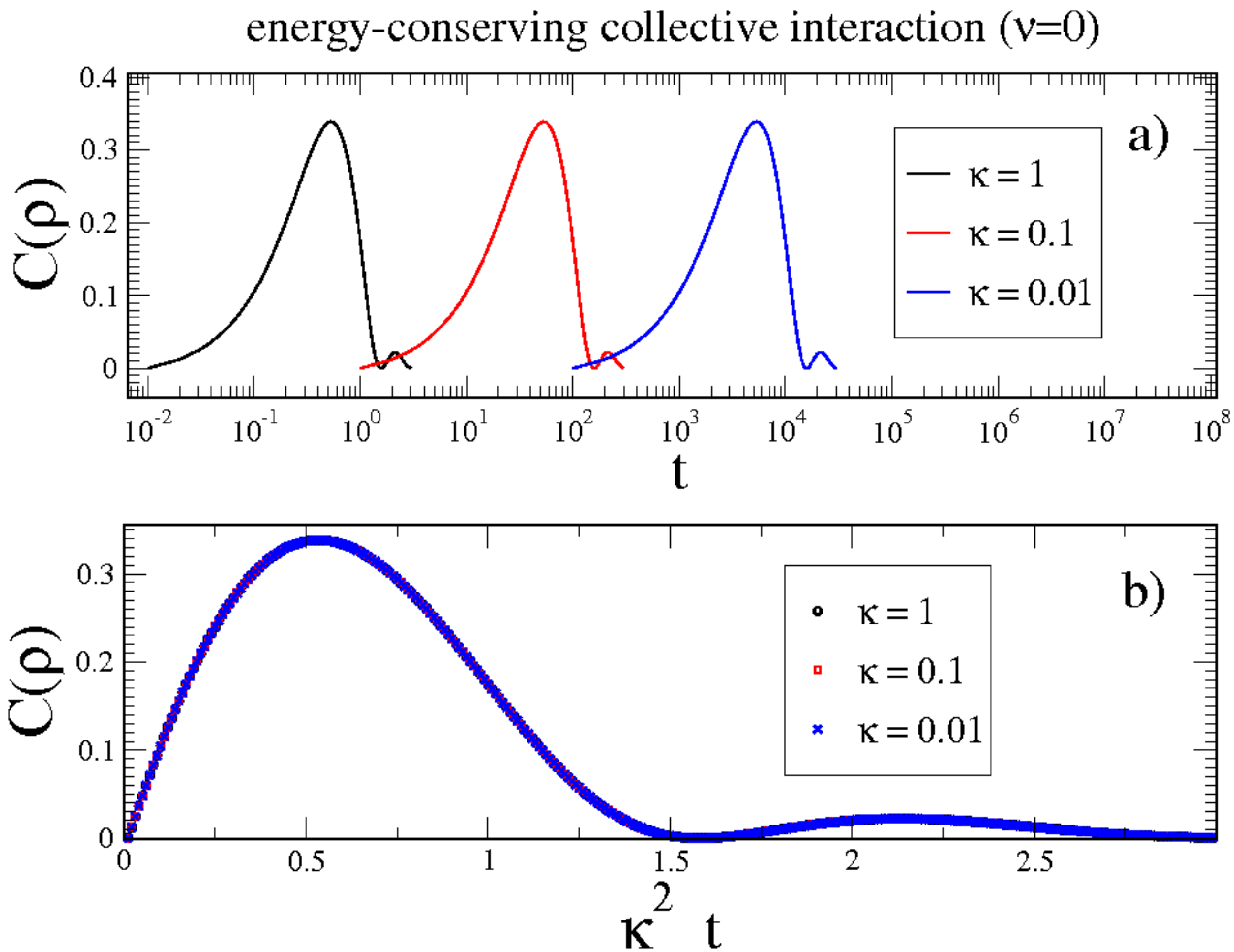
- *Purely energy-conserving coupling*

Evolution of the density matrix is not symmetric as function of κ (collective) and ν (local). Absence of collective coupling ($\kappa = 0$): concurrence evolution independent of local coupling; however for $\kappa \neq 0$ concurrence *depends* on ν (numerical results).

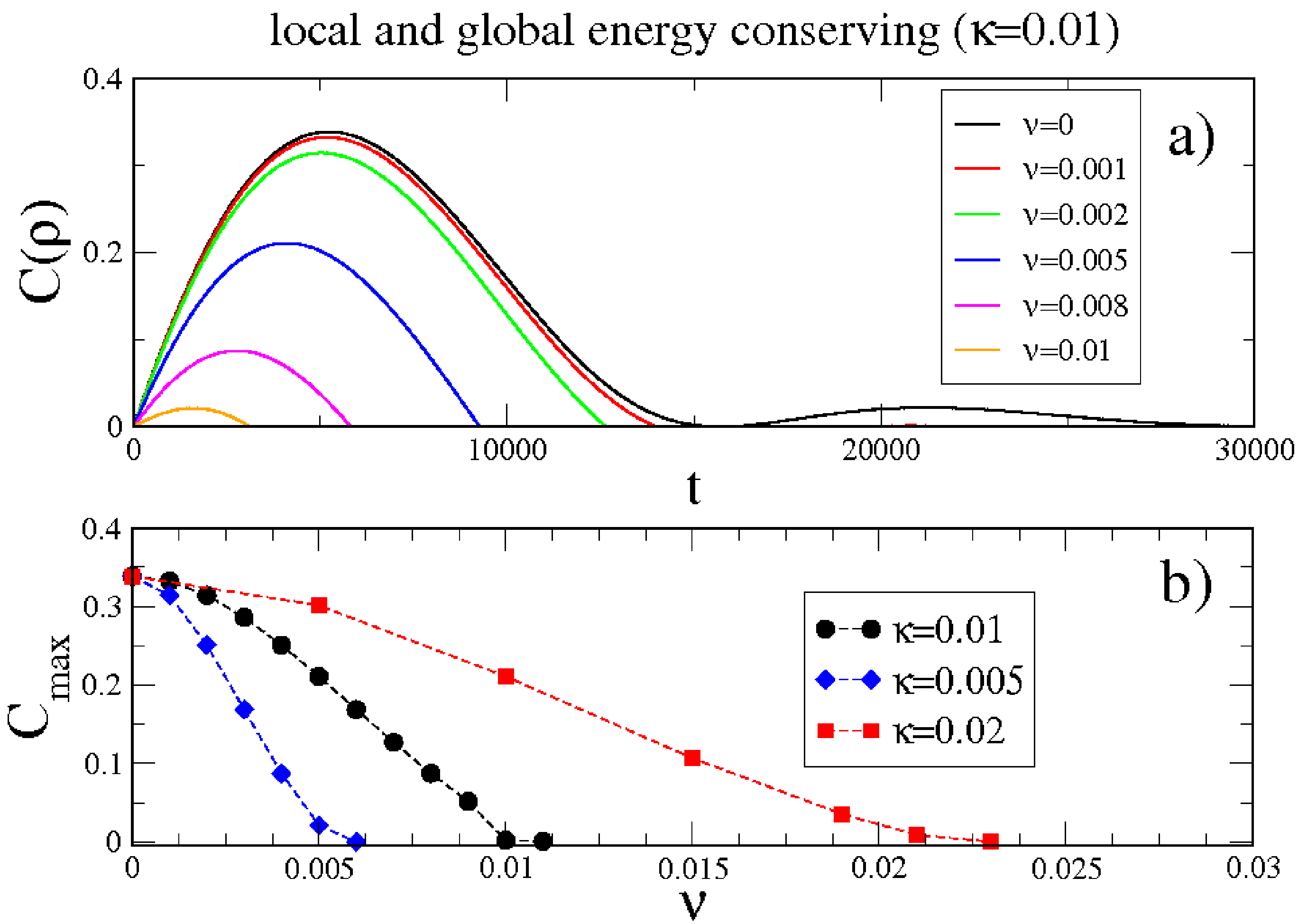
- *Full coupling*

Matrix elements evolve as complicated functions of all coupling parameters, showing that the effects of different interactions are correlated.

Numerical results: concurrence creation

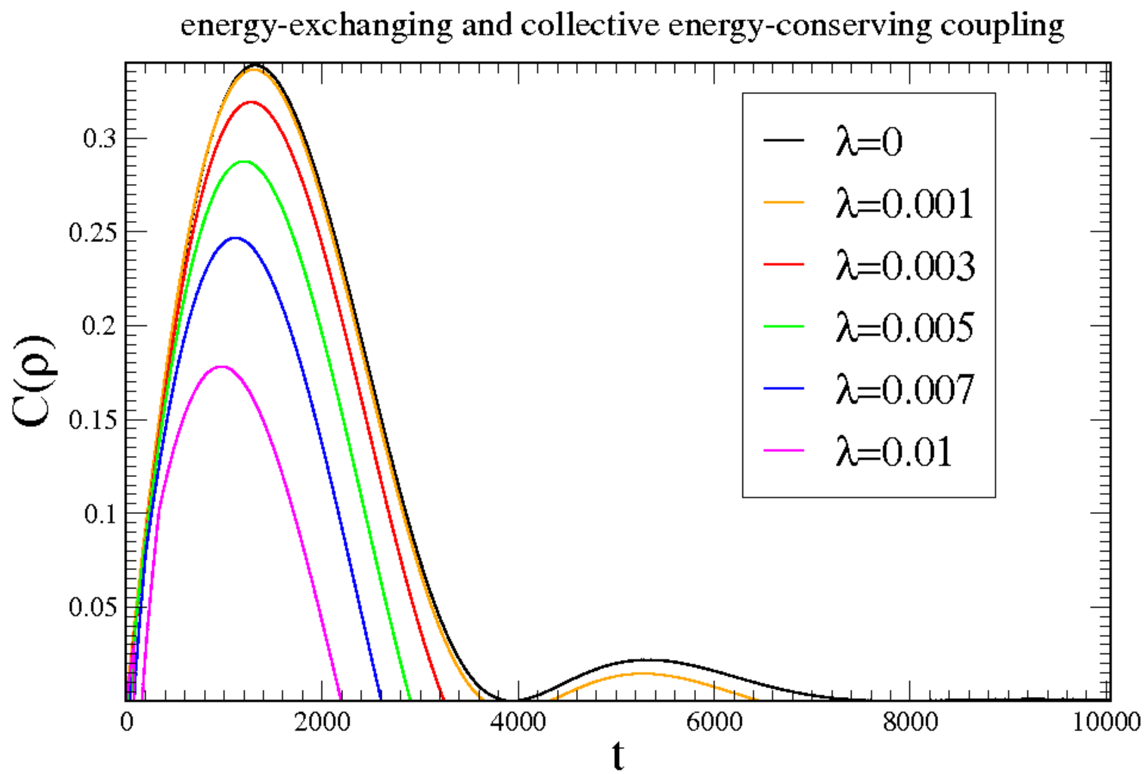


Amount of entanglement created is *independent* of coupling κ; peak at $t_0 \approx 0.5\kappa^{-2}$; revival of entanglement $t_1 \approx 2.1\kappa^{-2}$



Switching on local (energy conserving) coupling:

- creation of entanglement *reduced* (and delayed, $t_0 \propto (\kappa^2 + \nu^2)^{-1}$)
- local coupling exceeds collective one $\Rightarrow$ *no* concurrence is created



Energy-exchange collective and local interactions: $\lambda = \mu$ (symmetric); $\kappa = 0.02$ (collective, conserving), $\nu = 0$ fixed

• entanglement creation is *reduced* and peak time t_0 slightly reduced

• revival suppressed for increasing λ

• small times: density matrix in resonance approx. has partly negative eigenvalues (as Caldeira-Legget, Unruh-Zurek); numerics not reliable (non-smooth behavior in λ)

Conclusion

- New **resonance approach** to dynamics of open quantum systems:
  - Valid for all times $t \geq 0 \Rightarrow$ correct large-time behaviour
  - Cluster-wise independent markovian evolution $\Rightarrow$ different time scales $\Rightarrow$ simplification of analysis of quantum algorithms

- New results:
  - **Decoherence**:
    N qubits, collective energy conserving $+$ exchange coupling
    Decoherence rates: cons. $\propto N^2$, exch. $\propto N$, both: $+$ interference term
  - **Entanglement**:
    Two qubits, collective $+$ local, energy conserving $+$ exchange coupling
    Concurrence survival/death times in terms of cluster deco. times
    Numerical analysis of concurrence creation, sudden death, revival