Supplemental Material: Modeling of noise-assisted quantum transfer between donor and acceptor with finite bandwidths

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We present the technical details of our work. Starting from the Liouville equation, $i\dot{\rho} = [\tilde{\mathcal{H}}, \rho]$, we derive the dynamics of the average density matrix. We find the conditions of validity of the approximations that allow us to replace the exact dynamics (a set of coupled integro-differential equations) by rate-type differential equations. We use the interaction representation, viewing the off-diagonal elements as perturbations. The noise, $\xi(t)$, we consider is stationary, given by the random telegraph process (RTP). It is centered, $\langle\xi(t)\rangle = 0$, with correlation function, $\chi(t-t') = \langle\xi(t)\xi(t')\rangle$.

I. A SINGLE DONOR LEVEL AND A FINITE ACCEPTOR BANDWIDTH

We start with the simplified model consisting of a single electron energy level of the donor and an acceptor consisting of continuous band. Writing the Hamiltonian of the system as, $\tilde{\mathcal{H}} = \mathcal{H}_0 + W$, where

$$\begin{aligned}\mathcal{H}_0 \equiv \mathcal{H}_0(t) =& (E_0^{(d)} + \lambda_d(t))|d\rangle\langle d| + \int (E + \lambda_a(t))|E\rangle\langle E|\varrho(E)dE \\ W =& \int V|d\rangle\langle E|\varrho(E)dE + \text{h.c.},\end{aligned} \tag{1}$$

$\varrho(E)$ being the density of electron states of the acceptor, we define the interaction picture density matrix $\tilde{\rho}$ and operator $\tilde{W}$ by,

$$\tilde{\rho} = e^{i\int_0^t \mathcal{H}_0(\tau)d\tau}\rho\, e^{-i\int_0^t \mathcal{H}_0(\tau)d\tau}, \tag{2}$$

$$\tilde{W}(t) = e^{i\int_0^t \mathcal{H}_0(\tau)d\tau} W e^{-i\int_0^t \mathcal{H}_0(\tau)d\tau}. \tag{3}$$

A computation yields,

$$\tilde{V}(E,t) := \tilde{W}_{1E}(t) = V e^{i\varphi(t)} e^{i(E_0^{(d)} - E)t}, \tag{4}$$

$$\tilde{\rho}_{1E} = \rho_{1E} e^{i\varphi(t)} e^{i(E_0^{(d)} - E)t}, \tag{5}$$

$$\tilde{\rho}_{EE'} = \rho_{EE'} e^{i(E-E')t}, \tag{6}$$

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where $\varphi(t) = D\int_0^t \xi(t')dt'$ and $D = g_d - g_a$. The subscript 1 denotes the donor level, while the subscript E refers to the continuous energy levels. Note, that $\tilde\rho_{11}(t) = \rho_{11}(t)$ and $\tilde\rho_{EE}(t) = \rho_{EE}(t)$ (diagonal density matrix elements).

In the interaction representation, we obtain the following equations of motion:

$$\frac{d}{dt}\tilde\rho_{11} = i\int dE\varrho(E)(\tilde\rho_{1E}\tilde V^*(E,t) - \tilde V(E,t)\tilde\rho_{E1}), \tag{7}$$

$$\frac{d}{dt}\tilde\rho_{EE'} = i\tilde\rho_{E1}\tilde V(E',t) - i\tilde\rho_{1E'}\tilde V^*(E,t), \tag{8}$$

$$\frac{d}{dt}\tilde\rho_{1E} = i\tilde V(E,t)\rho_{11} - i\int dE'\varrho(E')\tilde V(E',t)\tilde\rho_{E'E}, \tag{9}$$

$$\frac{d}{dt}\tilde\rho_{E1} = -i\tilde V^*(E,t)\tilde\rho_{11} + i\int dE'\varrho(E')\tilde V^*(E',t)\tilde\rho_{EE'}. \tag{10}$$

We take the initial condition $\tilde\rho_{1E}(0) = \tilde\rho_{E1}(0) = 0$ for all E. Then, using Eqs. (7) - (10), we obtain,

$$\tilde\rho_{1E} = i\int_0^t dt'\Big(\tilde V(E,t')\tilde\rho_{11}(t') - \int dE'\varrho(E')\tilde V(E',t')\tilde\rho_{E'E}(t')\Big), \tag{11}$$

$$\tilde\rho_{E1} = -i\int_0^t dt'\Big(\tilde V^*(E,t')\tilde\rho_{11}(t') - \int dE'\varrho(E')\tilde V^*(E',t')\tilde\rho_{EE'}(t')\Big). \tag{12}$$

Now, inserting (11) and (12) into Eqs. (7) - (10), we obtain the following system of integro-differential equations,

$$\begin{aligned}\frac{d}{dt}\tilde\rho_{11}(t) = &-\int_0^t dt'\int dE\varrho(E)\Big(\tilde V^*(E,t)\tilde V(E,t') + \tilde V^*(E,t')\tilde V(E,t)\Big)\tilde\rho_{11}(t')\\ &+\int_0^t dt'\iint dEdE'\varrho(E)\varrho(E')\tilde V^*(E,t)\tilde V(E',t')\tilde\rho_{E'E}(t') + \tilde V(E,t)\tilde V^*(E',t')\tilde\rho_{EE'}(t'),\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{d}{dt}\tilde\rho_{22}(t)\rangle = &\int_0^t dt'\int dE\varrho(E)\Big(\tilde V^*(E,t)\tilde V(E,t') + \tilde V^*(E,t')\tilde V(E,t)\Big)\tilde\rho_{11}(t')\\ &-\int_0^t dt'\iint dEdE'\varrho(E)\varrho(E')\tilde V^*(E,t)\tilde V(E',t')\tilde\rho_{E'E}(t') + \tilde V(E,t)\tilde V^*(E',t')\tilde\rho_{EE'}(t'),\end{aligned} \tag{14}$$

$$\begin{aligned}\frac{d}{dt}\tilde\rho_{EE'}(t) = &\int_0^t dt'\Big(\tilde V^*(E,t')\tilde V(E',t) + \tilde V^*(E,t)\tilde V(E',t')\Big)\tilde\rho_{11}(t')\\ &-\int_0^t dt'\int dE''\varrho(E'')\tilde V^*(E,t)\tilde V(E'',t')\tilde\rho_{E''E'}(t') + \tilde V(E',t)\tilde V^*(E'',t')\tilde\rho_{EE''}(t').\end{aligned} \tag{15}$$

In this supplementary material, it is convenient to denote the average of the total acceptor probability by $\langle\tilde\rho_{22}(t)\rangle = \int\langle\tilde\rho_{EE}(t)\rangle\varrho(E)dE$. In the main text, we used the symbol $p_a(t)$ for this quantity. Averaging over the noise gives

$$\begin{aligned}\frac{d}{dt}\langle\tilde\rho_{11}(t)\rangle = &-\int_0^t dt'\int dE\varrho(E)\Big\langle\Big(\tilde V^*(E,t)\tilde V(E,t') + \tilde V^*(E,t')\tilde V(E,t)\Big)\tilde\rho_{11}(t')\Big\rangle\\ &+\int_0^t dt'\iint dEdE'\varrho(E)\varrho(E')\Big\langle\tilde V^*(E,t)\tilde V(E',t')\tilde\rho_{E'E}(t') + \tilde V(E,t)\tilde V^*(E',t')\tilde\rho_{EE'}(t')\Big\rangle,\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{d}{dt}\langle\tilde\rho_{22}(t)\rangle = &\int_0^t dt'\int dE\varrho(E)\Big\langle\Big(\tilde V^*(E,t)\tilde V(E,t') + \tilde V^*(E,t')\tilde V(E,t)\Big)\tilde\rho_{11}(t')\Big\rangle\\ &-\int_0^t dt'\iint dEdE'\varrho(E)\varrho(E')\Big\langle\tilde V^*(E,t)\tilde V(E',t')\tilde\rho_{E'E}(t') + \tilde V(E,t)\tilde V^*(E',t')\tilde\rho_{EE'}(t')\Big\rangle,\end{aligned} \tag{17}$$

$$\begin{aligned}\frac{d}{dt}\langle\tilde\rho_{EE'}(t)\rangle = &\int_0^t dt'\Big\langle\Big(\tilde V^*(E,t')\tilde V(E',t) + \tilde V^*(E,t)\tilde V(E',t')\Big)\tilde\rho_{11}(t')\Big\rangle\\ &-\int_0^t dt'\int dE''\varrho(E'')\Big\langle\tilde V^*(E,t)\tilde V(E'',t')\tilde\rho_{E''E'}(t') + \tilde V(E',t)\tilde V^*(E'',t')\tilde\rho_{EE''}(t')\Big\rangle.\end{aligned} \tag{18}$$

The terms in (16) - (18), containing products of the form $\tilde V^*(E,t)\tilde V(E,t')$, give rise to correlators of the form, $\big\langle e^{-i\varphi(t)}e^{-i\varphi(t')}\tilde\rho_{11}(t')\big\rangle$ (see (4)). To proceed further, we must split these correlators. Consider $\big\langle e^{i\varphi(t)}e^{-i\varphi(t')}\tilde\rho_{\alpha\beta}(t')\big\rangle$, where the indices α,β take values in $\{1,2,E,E'\}$. For the random telegraph process (RTP) we have (for details see Corollary 1 in Appendix A),

$$\big\langle e^{i\varphi(t)}e^{-i\varphi(t')}\tilde\rho_{\alpha\beta}(t')\big\rangle = \Phi(t-t')\big\langle\tilde\rho_{\alpha\beta}(t')\big\rangle - \frac{1}{iD\sigma}\frac{d}{dt'}\Phi(t-t')\big\langle\tilde\rho^{\xi}_{\alpha\beta}(t')\big\rangle, \tag{19}$$

where $\big\langle\tilde\rho^{\xi}_{\alpha\beta}(t')\big\rangle = (1/\sigma)\big\langle\xi(t')\tilde\rho_{\alpha\beta}(t')\big\rangle$, $\Phi(t-t') = \langle e^{i\varphi(t)}e^{-i\varphi(t')}\rangle$, and we set $\sigma^2 = \chi(0)$.

Using the differential formula (A8), we obtain,

$$\Big(\frac{d}{dt} + 2\gamma\Big)\big\langle\tilde\rho^{\xi}_{\alpha\beta}(t)\big\rangle = \Big\langle\xi(t)\frac{d}{dt}\tilde\rho_{\alpha\beta}(t)\Big\rangle. \tag{20}$$

The r.h.s. of this equation can be obtained for Eqs. (13) – (15) by multiplying both sides of these equations with $\xi(t)$ and then averaging over the random process. The result can be written as,

$$\Big\langle\xi(t)\frac{d}{dt}\tilde\rho_{\alpha\beta}(t)\Big\rangle = |V|^2\Psi_{\alpha\beta}\Big(\big\langle\tilde\rho_{\mu\nu}(t)\big\rangle, \big\langle\tilde\rho^{\xi}_{\mu\nu}(t)\big\rangle\Big), \tag{21}$$

where we denote by, $|V|^2\Psi_{\alpha\beta}\Big(\big\langle\tilde\rho_{\mu\nu}(t)\big\rangle, \big\langle\tilde\rho^{\xi}_{\mu\nu}(t)\big\rangle\Big)$, the result of the procedure described above. Then, Eq. (20) can be recast as,

$$\frac{d}{dt}\big\langle\tilde\rho^{\xi}_{\alpha\beta}(t)\big\rangle = -2\gamma\big\langle\tilde\rho^{\xi}_{\alpha\beta}(t)\big\rangle + |V|^2\Psi_{\alpha\beta}\Big(\big\langle\tilde\rho_{\mu\nu}(t)\big\rangle, \big\langle\tilde\rho^{\xi}_{\mu\nu}(t)\big\rangle\Big). \tag{22}$$

As one can see, the solution for $\big\langle\tilde\rho^{\xi}_{\alpha\beta}(t)\big\rangle$ can be written as,

$$\big\langle\tilde\rho^{\xi}_{\alpha\beta}(t)\big\rangle = |V|^2 e^{-2\gamma t}F_{\alpha\beta}(t), \tag{23}$$

where $F_{\alpha\beta}(t)$ obyes the following eqution:

$$\frac{d}{dt}F_{\alpha\beta}(t) = e^{2\gamma t}\Psi_{\alpha\beta}\Big(\big\langle\tilde\rho_{\mu\nu}(t)\big\rangle, \big\langle\tilde\rho^{\xi}_{\mu\nu}(t)\big\rangle\Big). \tag{24}$$

Thus, we conclude that Eq. (19) can be recast as,

$$\big\langle e^{i\varphi(t)}e^{-i\varphi(t')}\tilde\rho_{\alpha\beta}(t')\big\rangle = \Phi(t-t')\big\rangle\big\langle\tilde\rho_{\alpha\beta}(t')\big\rangle + \mathcal{O}(|V|^2). \tag{25}$$

In what follows, we neglect by higher terms, $\mathcal{O}(|V|^2)$, and write

$$\big\langle e^{i\varphi(t)}e^{-i\varphi(t')}\tilde\rho_{\alpha\beta}(t')\big\rangle \approx \Phi(t,t')\big\langle\tilde\rho_{\alpha\beta}(t')\big\rangle. \tag{26}$$

After splitting of correlations and neglecting the terms $\mathcal{O}(|V|^4)$ in Eqs. (16) – (18), we obtain for the average components of the density matrix the system of integro-differential equations:

$$\begin{aligned}\frac{d}{dt}\langle\tilde\rho_{11}(t)\rangle = &-\int_0^t dt'\int dE\varrho(E)\Big(\big\langle\tilde V^*(E,t)\tilde V(E,t')\big\rangle + \big\langle\tilde V^*(E,t')\tilde V(E,t)\big\rangle\Big)\big\langle\tilde\rho_{11}(t')\big\rangle\\ &+\int_0^t dt'\iint dEdE'\varrho(E)\varrho(E')\Big(\big\langle\tilde V^*(E,t)\tilde V(E',t')\big\rangle\big\langle\tilde\rho_{E'E}(t')\big\rangle + \big\langle\tilde V(E,t)\tilde V^*(E',t')\big\rangle\big\langle\tilde\rho_{EE'}(t')\big\rangle\Big),\end{aligned} \tag{27}$$

$$\begin{aligned}\frac{d}{dt}\langle\tilde\rho_{22}(t)\rangle = &\int_0^t dt'\int dE\varrho(E)\Big(\big\langle\tilde V^*(E,t)\tilde V(E,t')\big\rangle + \big\langle\tilde V^*(E,t')\tilde V(E,t)\big\rangle\Big)\big\langle\tilde\rho_{11}(t')\big\rangle\\ &-\int_0^t dt'\iint dEdE'\varrho(E)\varrho(E')\Big(\big\langle\tilde V^*(E,t)\tilde V(E',t')\big\rangle\big\langle\tilde\rho_{E'E}(t')\big\rangle + \big\langle\tilde V(E,t)\tilde V^*(E',t')\big\rangle\big\langle\tilde\rho_{EE'}(t')\big\rangle\Big),\end{aligned} \tag{28}$$

$$\begin{aligned}\frac{d}{dt}\langle\tilde\rho_{EE'}(t)\rangle = &\int_0^t dt'\Big(\big\langle\tilde V^*(E,t')\tilde V(E',t)\big\rangle + \big\langle\tilde V^*(E,t)\tilde V(E',t')\big\rangle\Big)\big\langle\tilde\rho_{11}(t')\big\rangle\\ &-\int_0^t dt'\int dE''\varrho(E'')\Big(\big\langle\tilde V^*(E,t)\tilde V(E'',t')\big\rangle\big\langle\tilde\rho_{E''E'}(t')\big\rangle + \big\langle\tilde V(E',t)\tilde V^*(E'',t')\big\rangle\big\langle\tilde\rho_{EE''}(t')\big\rangle\Big).\end{aligned} \tag{29}$$

Then, using Eqs. (27) - (29) and the definition $\tilde{V}(E,t) = V e^{i(E_0^{(d)}-E)t} e^{i\varphi(t)}$, we obtain

$$\begin{aligned}
\frac{d}{dt}\langle\tilde{\rho}_{11}(t)\rangle &= -2|V|^2 \mathrm{Re}\int_0^t dt' \Phi(t-t') \int dE \varrho(E) e^{i(E_0^{(d)}-E)(t-t')} \big\langle\tilde{\rho}_{11}(t')\big\rangle \\
&\quad +2|V|^2 \mathrm{Re}\int_0^t dt' \Phi(t-t') \iint dE dE' \varrho(E)\varrho(E') e^{i(E_0^{(d)}-E)t} e^{-i(E_0^{(d)}-E')t'} \big\langle\tilde{\rho}_{EE'}(t')\big\rangle, \qquad (30)\\
\frac{d}{dt}\langle\tilde{\rho}_{22}(t)\rangle &= 2|V|^2 \mathrm{Re}\int_0^t dt' \Phi(t-t') \int dE \varrho(E) e^{i(E_0^{(d)}-E)(t-t')} \big\langle\tilde{\rho}_{11}(t')\big\rangle \\
&\quad -2|V|^2 \mathrm{Re}\int_0^t dt' \Phi(t-t') \iint dE dE' \varrho(E)\varrho(E') e^{i(E_0^{(d)}-E)t} e^{-i(E_0^{(d)}-E')t'} \big\langle\tilde{\rho}_{EE'}(t')\big\rangle, \qquad (31)\\
\frac{d}{dt}\langle\tilde{\rho}_{EE'}(t)\rangle &= |V|^2 \int_0^t dt' \Big[\Phi(t-t') e^{-i(E_0^{(d)}-E)t'} e^{i(E_0^{(d)}-E')t} + \Phi(t'-t) e^{-i(E_0^{(d)}-E)t} e^{i(E_0^{(d)}-E')t'}\Big] \big\langle\tilde{\rho}_{11}(t')\big\rangle \\
&\quad -|V|^2 \int_0^t dt' \Big[\Phi(t-t') \int dE'' \varrho(E'') e^{i(E_0^{(d)}-E')t} e^{-i(E_0^{(d)}-E'')t'} \big\langle\tilde{\rho}_{EE''}(t')\big\rangle \\
&\qquad + \Phi(t'-t) \int dE'' \varrho(E'') e^{-i(E_0^{(d)}-E)t} e^{i(E_0^{(d)}-E'')t'} \big\langle\tilde{\rho}_{E''E'}(t')\big\rangle\Big], \qquad (32)
\end{aligned}$$

where $\Phi(t-t') = \Big\langle \exp\{iD \int\limits_0^{t-t'} \xi(\tau)d\tau\}\Big\rangle$.

In what follows, we consider a Gaussian density of states in the acceptor band, centered at some point, $E_0^{(a)}$,

$$\varrho(E) = \varrho_0 e^{-\alpha\left(E-E_0^{(a)}\right)^2}. \qquad (33)$$

Then we compute

$$\langle\tilde{\rho}_{22}(t')\rangle = \int \langle\tilde{\rho}_{EE}(t')\rangle \varrho(E) dE \approx \langle\tilde{\rho}_{E_0^{(a)}E_0^{(a)}}(t')\rangle \varrho_0 \sqrt{\tfrac{\pi}{\alpha}}, \qquad (34)$$

$$\int dE \varrho(E) e^{i(E_0^{(d)}-E)(t-t')} = \varrho_0 \sqrt{\tfrac{\pi}{\alpha}} e^{i\varepsilon(t-t')} e^{-\frac{(t-t')^2}{4\alpha}}, \qquad (35)$$

and

$$\begin{aligned}
&\iint dE dE' \varrho(E)\varrho(E') e^{i(E_0^{(d)}-E)t} e^{-i(E_0^{(d)}-E')t'} \big\langle\tilde{\rho}_{EE'}(t')\big\rangle \\
&\approx \Big(\varrho_0 \sqrt{\frac{\pi}{\alpha}} \langle\tilde{\rho}_{E_0^{(a)}E_0^{(a)}}(t')\rangle + \frac{i}{2\alpha} t' \langle\tilde{X}(t')\rangle - \frac{i}{2\alpha} t \langle\tilde{X}^*(t')\rangle\Big) \varrho_0 \sqrt{\frac{\pi}{\alpha}} e^{i\varepsilon(t-t')} e^{-\frac{t^2+t'^2}{4\alpha}} \\
&\approx \Big(\langle\tilde{\rho}_{22}(t')\rangle + \frac{i}{2\alpha} t' \langle\tilde{X}(t')\rangle - \frac{i}{2\alpha} t \langle\tilde{X}^*(t')\rangle\Big) \varrho_0 \sqrt{\frac{\pi}{\alpha}} e^{i\varepsilon(t-t')} e^{-\frac{t^2+t'^2}{4\alpha}}, \qquad (36)
\end{aligned}$$

where $\varepsilon = E_0^{(d)} - E_0^{(a)}$ is the difference between the donor energy and the center of the acceptor energy band and,

$$\langle\tilde{X}(t')\rangle = \varrho_0 \sqrt{\frac{\pi}{\alpha}} \frac{\partial}{\partial E'} \langle\tilde{\rho}_{EE'}(t')\rangle\Big|_{E=E'=E_0^{(a)}}. \qquad (37)$$

Using these results and Eq. (30), we obtain the following integro-differential equations:

$$\begin{aligned}
\frac{d}{dt}\langle\tilde{\rho}_{11}(t)\rangle &= -2\mathrm{Re}\int_0^t dt' \tilde{K}_1(t,t') \big\langle\tilde{\rho}_{11}(t')\big\rangle + 2\mathrm{Re}\int_0^t dt' \tilde{K}_2(t,t') \big\langle\tilde{\rho}_{22}(t')\big\rangle \\
&\quad -\frac{1}{\alpha}\mathrm{Im}\int_0^t dt' \tilde{K}_2(t,t') \big(t'\langle\tilde{X}(t')\rangle - t\langle\tilde{X}^*(t')\rangle\big), \qquad (38)\\
\frac{d}{dt}\langle\tilde{\rho}_{22}(t)\rangle &= 2\mathrm{Re}\int_0^t dt' \tilde{K}_1(t,t') \big\langle\tilde{\rho}_{11}(t')\big\rangle - 2\mathrm{Re}\int_0^t dt' \tilde{K}_2(t,t') \big\langle\tilde{\rho}_{22}(t')\big\rangle \\
&\quad +\frac{1}{\alpha}\mathrm{Im}\int_0^t dt' \tilde{K}_2(t,t') \big(t'\langle\tilde{X}(t')\rangle - t\langle\tilde{X}^*(t')\rangle\big), \qquad (39)
\end{aligned}$$

where

$$\tilde{K}_1(t,t') = v^2\Phi(t-t')\exp\Big(i\varepsilon(t-t') - \frac{(t-t')^2}{4\alpha}\Big), \quad (40)$$

$$\tilde{K}_2(t,t') = v^2\Phi(t-t')\exp\Big(i\varepsilon(t-t') - \frac{t^2+t'^2}{4\alpha}\Big). \quad (41)$$

Here we denote $v^2 = |V|^2\varrho_0\sqrt{\pi/\alpha}$. Next, we expand,

$$\langle\tilde{\rho}_{11}(t')\rangle \approx \langle\rho_{11}(t)\rangle - (t-t')\frac{d}{dt}\langle\rho_{11}(t)\rangle. \quad (42)$$

Using this expansion and neglecting the higher order terms, we find that (39) can be approximated by the ordinary differential equations,

$$\frac{d}{dt}\langle\rho_{11}\rangle = -\Re_1(t)\langle\rho_{11}\rangle - D\sigma\frac{\partial}{\partial\varepsilon}\Re_1(t)\langle\rho^\xi_{11}\rangle + \Re_2(t)\langle\rho_{22}\rangle + D\sigma\frac{\partial}{\partial\varepsilon}\Re_2(t)\langle\rho^\xi_{22}\rangle, \quad (43)$$

$$\frac{d}{dt}\langle\rho_{22}\rangle = -\Re_2(t)\langle\rho_{22}\rangle - D\sigma\frac{\partial}{\partial\varepsilon}\Re_2(t)\langle\rho^\xi_{22}\rangle + \Re_1(t)\langle\tilde{\rho}_{11}\rangle + D\sigma\frac{\partial}{\partial\varepsilon}\Re_1(t)\langle\rho^\xi_{11}\rangle, \quad (44)$$

$$\frac{d}{dt}\langle\rho^\xi_{11}\rangle = -2\gamma\langle\rho^\xi_{11}\rangle - \Re_1(t)\langle\rho^\xi_{11}\rangle - D\sigma\frac{\partial}{\partial\varepsilon}\Re_1(t)\langle\rho_{11}\rangle + \Re_2(t)\langle\rho^\xi_{22}\rangle + D\sigma\frac{\partial}{\partial\varepsilon}\Re_2(t)\langle\rho_{22}\rangle, \quad (45)$$

$$\frac{d}{dt}\langle\rho^\xi_{22}\rangle = -2\gamma\langle\rho^\xi_{22}\rangle - \Re_2(t)\langle\rho^\xi_{22}\rangle - D\sigma\frac{\partial}{\partial\varepsilon}\Re_2(t)\langle\rho_{22}\rangle + \Re_1(t)\langle\tilde{\rho}^\xi_{11}\rangle + D\sigma\frac{\partial}{\partial\varepsilon}\Re_1(t)\langle\rho_{11}\rangle, \quad (46)$$

$$(47)$$

where $\Re_{1,2}(t) = 2\mathrm{Re}\int_0^t \tilde{K}_{1,2}(t,t')dt'$, $\langle\rho_{11}\rangle + \langle\rho_{22}\rangle = 1$ and $\langle\rho^\xi_{11}(0)\rangle = \langle\rho^\xi_{22}(0)\rangle = 0$.

Our numerical simulations show that if the conditions of validity, (58) given below, are satisfied, then one can simplify this system as follows:

$$\frac{d}{dt}\langle\rho_{11}\rangle = -\Re_1(t)\langle\rho_{11}\rangle + \Re_2(t)\langle\rho_{22}\rangle, \quad (48)$$

$$\frac{d}{dt}\langle\rho_{22}\rangle = \Re_1(t)\langle\tilde{\rho}_{11}\rangle - \Re_2(t)\langle\rho_{22}\rangle. \quad (49)$$

The solution of these equations with the initial condition $\langle\rho_{11}(0)\rangle = 1$ is

$$\langle\rho_{11}(t)\rangle = e^{-f(t)}\big(1 + \int_0^t \Re_2(s)e^{f(s)}ds\big), \quad (50)$$

$$\langle\rho_{22}(t)\rangle = 1 - \langle\rho_{11}(t)\rangle, \quad (51)$$

where $f(t) = \int_0^t \big(\Re_1(\tau) + \Re_2(\tau)\big)d\tau$.

To proceed further, one needs to know the explicit expression for the characteristic functional $\Phi(t-t')$. Using (26), we find that $\Phi(t)$ obeys the following integro-differential equation [1, 2]:

$$\frac{d}{dt}\Phi(t) = -D^2\int_0^t \chi(t-t')\Phi(t')dt'. \quad (52)$$

One can show that in the time interval, $0 < t < \infty$, the Gaussian approximation is valid yielding,

$$\frac{d}{dt}\Phi(t) \approx -\Phi(t)D^2\int_0^t \chi(t-t')dt', \quad (53)$$

if $|D^2\int_0^\infty t\chi(t)dt| \ll 1$. This condition can be recast as follows: $D\sqrt{\chi(0)} \ll 1/\tau_c$, where τ_c is the correlation time [3],

$$\tau_c = \frac{1}{\chi(0)}\int_0^\infty \chi(t)dt. \quad (54)$$

The solution of Eq. (53) can be written as,

$$\Phi(t) = \exp\Big(-D^2\int_0^t dt'\int_0^{t'} dt''\chi(t'-t'')\Big). \quad (55)$$

To obtain the analytic expressions for the dynamical rates, $\mathfrak{R}_{1,2}(t) = 2\mathrm{Re}\int_0^t \tilde{K}_{1,2}(t,t')dt'$, we write

$$\Phi(t) = \exp\Big(-\frac{D^2\chi(0)}{2}t^2 - \frac{D^2\dot{\chi}(0)}{3!}t^3 + \dots\Big). \tag{56}$$

If $|t_0 d\ln\chi(0/dt)| \ll 3$, where $t_0 = i2\varepsilon/p^2$, in computation of the dynamical rate one can neglect the higher orders in $\Phi(t)$ and use

$$\Phi(t) = \exp\Big(-\frac{D^2\chi(0)}{2}t^2\Big). \tag{57}$$

Finally, gathering all results together, we find that the Gaussian approximation in its simplified form, (57), can be used for computation of the dynamical rates, if the following conditions hold:

$$|D\sqrt{\chi(0)}| \ll \left|\frac{d}{dt}\ln\chi(t)\Big|_{t=0}\right| \ll \frac{3p^2}{2\varepsilon}, \tag{58}$$

where $p = \sqrt{1/\alpha + 2D^2\chi(0)}$. The parameter p includes contributions of the acceptor bandwidth and the intensity of noise. Next, employing Eqs. (40), (41) and (57), we obtain

$$\mathfrak{R}_1(t) = \frac{\sqrt{\pi}\,v^2}{p}\exp\Big(-\frac{\varepsilon^2}{p^2}\Big)\Big(\mathrm{erf}\Big(\frac{pt}{2}+i\frac{\varepsilon}{p}\Big)+\mathrm{erf}\Big(\frac{pt}{2}-i\frac{\varepsilon}{p}\Big)\Big), \tag{59}$$

$$\mathfrak{R}_2(t) = \frac{\sqrt{\pi}\,v^2}{p}\Big\{\exp\Big(-\frac{t^2}{2\alpha}+\frac{(t/2\alpha - i\varepsilon)^2}{p^2}\Big)\Big(\mathrm{erf}\Big(\frac{pt}{2}-\frac{(t/2\alpha - i\varepsilon)}{p}\Big)+\mathrm{erf}\Big(\frac{(t/2\alpha - i\varepsilon)}{p}\Big)\Big)+\mathrm{c.c.}\Big\}, \tag{60}$$

where $\mathrm{erf}(z) = (2/\sqrt{\pi})\int_0^z e^{-t^2}dt$ is the error function [5].

A. Conditions for validity of the approximation

In this Section, we estimate the validity of the approximation leading to the Eqs. (48) and (49):

$$\frac{d}{dt}\langle\rho_{11}\rangle = -\mathfrak{R}_1(t)\langle\rho_{11}\rangle + \mathfrak{R}_2(t)\langle\rho_{22}\rangle, \tag{61}$$

$$\frac{d}{dt}\langle\rho_{22}\rangle = \mathfrak{R}_1(t)\langle\tilde{\rho}_{11}\rangle - \mathfrak{R}_2(t)\langle\rho_{22}\rangle. \tag{62}$$

We start with Eqs. (38) and (39) written as,

$$\frac{d}{dt}\langle\tilde{\rho}_{11}(t)\rangle = -\int_0^t K_1(t,t')\langle\tilde{\rho}_{11}(t')\rangle dt' + \int_0^t K_2(t,t')\langle\tilde{\rho}_{22}(t')\rangle dt' + \zeta(t), \tag{63}$$

$$\frac{d}{dt}\langle\tilde{\rho}_{22}(t)\rangle = \int_0^t K_1(t,t')\langle\tilde{\rho}_{11}(t')\rangle dt' - \int_0^t K_2(t,t')\langle\tilde{\rho}_{22}(t')\rangle dt' - \zeta(t), \tag{64}$$

where $\zeta(t) = \tilde{\zeta}(t) + \tilde{\zeta}^*(t)$, and

$$\tilde{\zeta}(t) = i\int_0^t \tilde{K}_2(t,t')\Big(\frac{t'}{2\alpha}\langle\tilde{X}^*(t')\rangle - \frac{t}{2\alpha}\langle\tilde{X}(t')\rangle\Big)dt'. \tag{65}$$

The solution of the Eqs. (63) and (64) can be written as, $\langle\tilde{\rho}_{11}(t)\rangle = \langle\rho_{11}(t)\rangle + \Delta\rho(t)$ and $\langle\tilde{\rho}_{22}(t)\rangle = \langle\rho_{22}(t)\rangle - \Delta\rho(t)$, where

$$\Delta\rho(t) = \int_0^t \zeta(t')dt', \tag{66}$$

and $\langle\rho_{11}(t)\rangle$ and $\langle\rho_{22}(t)\rangle$ obey the system of the non-perturbative integro-differential equations:

$$\frac{d}{dt}\langle\rho_{11}(t)\rangle = -\int_0^t K_1(t,t')\langle\rho_{11}(t')\rangle dt' + \int_0^t K_2(t,t')\langle\rho_{22}(t')\rangle dt', \tag{67}$$

$$\frac{d}{dt}\langle\rho_{22}(t)\rangle = \int_0^t K_1(t,t')\langle\rho_{11}(t')\rangle dt' - \int_0^t K_2(t,t')\langle\rho_{22}(t')\rangle dt'. \tag{68}$$

From here it follows that if

$$\frac{|\Delta\rho(t)|}{\langle\rho_{22}(t)\rangle} \ll 1, \quad 0 < t < \infty, \tag{69}$$

then Eqs. (63) and (64) can be replaced by Eqs. (67) and (68).

Employing Eqs. (32) and (61) and (62), we find that $\langle\tilde{X}(t)\rangle$ obeys the equation:

$$\frac{d}{dt}\langle\tilde{X}(t)\rangle = -i\int_0^t \big(t\tilde{K}_1(t,t') + t'\tilde{K}_1^*(t,t')\big)\langle\rho_{11}(t')\rangle dt' + it\int_0^t \tilde{K}_2(t,t')\langle\rho_{22}(t')\rangle dt'. \tag{70}$$

Next, expanding $\langle\rho_{11}(t')\rangle$ and $\langle\rho_{22}(t')\rangle$ as,

$$\langle\rho_{11}(t')\rangle \approx \langle\rho_{11}(t)\rangle - \frac{d}{dt}\langle\rho_{11}(t)\rangle(t-t'), \tag{71}$$

$$\langle\rho_{22}(t')\rangle \approx \langle\rho_{22}(t)\rangle - \frac{d}{dt}\langle\tilde{\rho}_{22}(t)\rangle(t-t'), \tag{72}$$

we find that equations (67) and (68) can be recast as,

$$\frac{d}{dt}\langle\rho_{11}\rangle = -\mathfrak{R}_1(t)(1+Y_1(t))\langle\rho_{11}\rangle + \mathfrak{R}_2(t)(1+Y_2(t))\langle\rho_{22}\rangle, \tag{73}$$

$$\frac{d}{dt}\langle\rho_{22}\rangle = \mathfrak{R}_1(t)(1+Y_1(t))\langle\tilde{\rho}_{11}\rangle - \mathfrak{R}_2(t)(1+Y_2(t))\langle\rho_{22}\rangle, \tag{74}$$

where $Y_{1,2}(t) = \int_0^t (t-t')K_{1,2}(t,t')dt'$. The latter can be rewritten as, $Y_{1,2}(t) = i(\partial\tilde{\mathfrak{R}}^*_{1,2}(t)/\partial\varepsilon - \partial\tilde{\mathfrak{R}}_{1,2}(t)/\partial\varepsilon)$. If

$$\Big|\int_0^t (t-t')K_{1,2}(t,t')dt'\Big| \ll 1, \quad t \in (0,\infty), \tag{75}$$

one can neglect the contribution of the terms, $Y_{1,2}(t)$, and the system of integro-differential equations (73) and (74) can be approximated by the following pair of ordinary differential equations,

$$\frac{d}{dt}\langle\rho_{11}\rangle = -\mathfrak{R}_1(t)\langle\tilde{\rho}_{11}\rangle + \mathfrak{R}_2(t)\langle\rho_{22}\rangle, \tag{76}$$

$$\frac{d}{dt}\langle\rho_{22}\rangle = \mathfrak{R}_1(t)\langle\tilde{\rho}_{11}\rangle - \mathfrak{R}_2(t)\langle\rho_{22}\rangle. \tag{77}$$

In the same order of expansion as above, we have

$$\frac{d}{dt}\langle\tilde{X}(t)\rangle = -it\mathfrak{R}_1(t)\langle\rho_{11}(t)\rangle - \frac{\partial\tilde{\mathfrak{R}}_1^*(t)}{\partial\varepsilon}\langle\rho_{11}(t)\rangle + it\tilde{\mathfrak{R}}_2(t)\langle\rho_{22}(t)\rangle, \tag{78}$$

$$\frac{d}{dt}\Delta\rho(t) = -\frac{1}{2\alpha}\frac{\partial\tilde{\mathfrak{R}}_2(t)}{\partial\varepsilon}\langle\tilde{X}^*(t)\rangle - \frac{it}{2\alpha}\tilde{\mathfrak{R}}_2(t)\big(\langle\tilde{X}(t)\rangle - \langle\tilde{X}^*(t)\rangle\big) + \text{c.c.}, \tag{79}$$

where

$$\tilde{\mathfrak{R}}_1(t) = \frac{\sqrt{\pi}\,v^2}{p}\exp\Big(-\frac{\varepsilon^2}{p^2}\Big)\Big(\mathrm{erf}\Big(\frac{pt}{2} + i\frac{\varepsilon}{p}\Big) - \mathrm{erf}\Big(i\frac{\varepsilon}{p}\Big)\Big), \tag{80}$$

$$\tilde{\mathfrak{R}}_2(t) = \frac{\sqrt{\pi}\,v^2}{p}\exp\Big(-\frac{t^2}{2\alpha} + \frac{(t/2\alpha - i\varepsilon)^2}{p^2}\Big)\Big(\mathrm{erf}\Big(\frac{pt}{2} - \frac{(t/2\alpha - i\varepsilon)}{p}\Big) + \mathrm{erf}\Big(\frac{(t/2\alpha - i\varepsilon)}{p}\Big)\Big). \tag{81}$$

Combining all results, one can see that in the time interval, $0 < t < \infty$, the original system of integro-differential equations (38) and (39) can be approximated by the system of ordinary differential equations, (76) and (77), provided

$$\Big|\Big(\frac{\partial\tilde{\mathfrak{R}}_{1,2}(t)}{\partial\varepsilon} - \frac{\partial\tilde{\mathfrak{R}}^*_{1,2}(t)}{\partial\varepsilon}\Big)\Big| \ll 1, \quad \text{and} \quad \frac{|\Delta\rho(t)|}{\langle\rho_{22}(t)\rangle} \ll 1. \tag{82}$$

To proceed further, it is convenient to define a scaled time, $\tau = pt$, and the dimensionless, complex dynamical rates, $\tilde{\mathfrak{r}}_{1,2}(\tau) = (1/p)\mathfrak{R}_{1,2}(\tau)$:

$$\tilde{\mathfrak{r}}_1(\tau) =\sqrt{\pi}a^2 \exp(-\eta^2)\Big(\mathrm{erf}\Big(\frac{\tau}{2} + i\eta\Big) - \mathrm{erf}\big(i\eta\big)\Big), \tag{83}$$

$$\tilde{\mathfrak{r}}_2(\tau) =\sqrt{\pi}a^2 \exp\Big(-\frac{\mu^2\tau^2}{2} + \Big(\frac{\mu\tau}{2} - i\eta\Big)^2\Big)\bigg(\mathrm{erf}\Big(\frac{\tau}{2} - \frac{\mu\tau}{2} + i\eta\Big) + \mathrm{erf}\Big(\frac{\mu\tau}{2} - i\eta\Big)\bigg), \tag{84}$$

where $a = v/p$, $\eta = \varepsilon/p$ and $\mu = 1/\sqrt{\alpha}p$.

Then, one can rewrite the conditions of Eq. (82) as,

$$a^2 \ll W_{1,2}(\tau;\eta,\mu) \quad \text{and} \quad Z(\tau;a,\eta,\mu) \ll 1, \quad 0 < \tau < \infty, \tag{85}$$

where

$$W_{1,2}(\tau;\eta,\mu) =\frac{a^2}{\left|\dfrac{\partial\tilde{\mathfrak{r}}_{1,2}(\tau)}{\partial\eta} - \dfrac{\partial\tilde{\mathfrak{r}}^*_{1,2}(\tau)}{\partial\eta}\right|}, \tag{86}$$

$$Z(\tau;a,\eta,\mu) =\frac{|\Delta\rho(\tau)|}{\langle\rho_{22}(\tau)\rangle} = \frac{\mu^2}{\langle\rho_{22}(\tau)\rangle}\left|\mathrm{Re}\int_0^\tau d\tau'\bigg(\frac{\partial\tilde{\mathfrak{r}}_2(\tau')}{\partial\eta}\langle\tilde{X}^*(\tau')\rangle + i\tau'\tilde{\mathfrak{r}}_2(\tau')\big(\langle\tilde{X}(\tau')\rangle - \langle\tilde{X}^*(\tau')\rangle\big)\bigg)\right|. \tag{87}$$

Now Eqs. (76) – (78) can be rewritten as,

$$\frac{d}{d\tau}\langle\rho_{11}(\tau)\rangle = -\,\mathfrak{r}_1(\tau)\big\langle\rho_{11}(\tau)\big\rangle + \mathfrak{r}_2(\tau)\big\langle\rho_{22}(\tau)\big\rangle, \tag{88}$$

$$\frac{d}{d\tau}\langle\rho_{22}(\tau)\rangle =\mathfrak{r}_1(\tau)\big\langle\rho_{11}(\tau)\big\rangle - \mathfrak{r}_2(\tau)\big\langle\rho_{22}(\tau)\big\rangle, \tag{89}$$

$$\frac{d}{d\tau}\langle\tilde{X}(\tau)\rangle = -\,i\tau\mathfrak{r}_1(\tau)\big\langle\rho_{11}(\tau)\big\rangle - \frac{\partial\tilde{\mathfrak{r}}^*_1(\tau)}{\partial\eta}\big\langle\rho_{11}(\tau)\big\rangle + i\tau\tilde{\mathfrak{r}}_2(\tau)\big\langle\rho_{22}(\tau)\big\rangle, \tag{90}$$

$$\frac{d}{d\tau}\Delta\rho(\tau) = -\frac{1}{2\alpha}\frac{\partial\tilde{\mathfrak{r}}_2(\tau)}{\partial\eta}\langle\tilde{X}^*(\tau)\rangle - \frac{i\tau}{2\alpha}\tilde{\mathfrak{z}}_2(\tau)\big(\langle\tilde{X}(\tau)\rangle - \langle\tilde{X}^*(\tau)\rangle\big) + \text{c.c.}, \tag{91}$$

where we set $\mathfrak{r}_{1,2} = 2\mathrm{Re}\,\tilde{\mathfrak{r}}_{1,2}$.

Let us denote the real and imaginary parts of $\langle\tilde{X}(\tau)\rangle$ as, $U(\tau) = \mathrm{Re}\langle\tilde{X}(\tau)\rangle$ and $V(\tau) = \mathrm{Im}\langle\tilde{X}(\tau)\rangle$. Then, employing Eq. (91), one can show that the functions $U(\tau)$ and $V(\tau)$ obey the following differential equations:

$$\frac{d}{d\tau}U(\tau) = -\,\mathrm{Re}\frac{\partial\tilde{\mathfrak{r}}_1(\tau)}{\partial\eta}\big\langle\rho_{11}(\tau)\big\rangle - \tau\mathrm{Im}\,\tilde{\mathfrak{r}}_2(\tau)\big\langle\rho_{22}(\tau)\big\rangle, \tag{92}$$

$$\frac{d}{d\tau}V(\tau) =\Big(\mathrm{Im}\frac{\partial\tilde{\mathfrak{r}}_1(\tau)}{\partial\eta} - \tau\mathfrak{r}_1(\tau)\Big)\big\langle\rho_{11}(\tau)\big\rangle + \tau\mathrm{Re}\,\tilde{\mathfrak{r}}_2(\tau)\big\langle\rho_{22}(\tau)\big\rangle. \tag{93}$$

In addition we obtain

$$|\Delta\rho(\tau;a,\eta,\mu)| =\mu^2\left|\int_0^\tau d\tau'\bigg(\mathrm{Re}\frac{\partial\tilde{\mathfrak{r}}_2(\tau')}{\partial\eta}U(\tau') + \mathrm{Im}\frac{\partial\tilde{\mathfrak{r}}_2(\tau')}{\partial\eta}V(\tau') - \tau'\mathfrak{r}_2(\tau')V(\tau')\bigg)\right|. \tag{94}$$

A straightforward computation shows that $|W_{1,2}(\tau;\mu,\eta)| \geq W(\eta)$, where (See Fig. 1)

$$W(\eta) = \frac{1}{|\mathrm{Re}(1 - \mathrm{i}\sqrt{\pi}\eta \mathrm{e}^{-\eta^2}\mathrm{erfc}(\mathrm{i}\eta))|} \geq 1. \tag{95}$$

Thus, the first condition from Eq. (85) can be replaced by the stronger inequality

$$\frac{v}{p} \ll 1. \tag{96}$$

Gathering all results, we conclude that in the time interval, $0 < t < \infty$, the original system of integro-differential

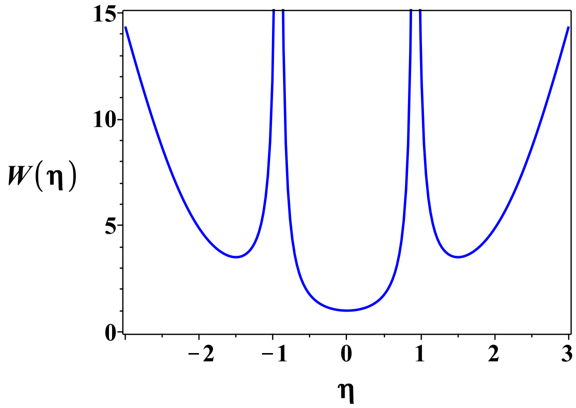

FIG. 1: (Color online) Dependence of W on η.

equations (38 and (39) can be approximated by the system of the rate-type ordinary differential equations:

$$\frac{d}{dt}\langle\rho_{11}\rangle = -\mathfrak{R}_1(t)\langle\tilde{\rho}_{11}\rangle + \mathfrak{R}_2(t)\langle\rho_{22}\rangle, \tag{97}$$

$$\frac{d}{dt}\langle\rho_{22}\rangle = \mathfrak{R}_1(t)\langle\tilde{\rho}_{11}\rangle - \mathfrak{R}_2(t)\langle\rho_{22}\rangle, \tag{98}$$

if the following conditions hold:

$$|D\sqrt{\chi(0)}| \ll \frac{d}{dt}\ln\chi(t)\Big|_{t=0} \ll \frac{3p^2}{2\varepsilon}, \tag{99}$$

$$\frac{v}{p} \ll 1, \quad \text{and} \quad \frac{|\Delta\rho(t)|}{\langle\rho_{22}(t)\rangle} \ll 1. \tag{100}$$

II. FINITE ELECTRON BANDS

The Hamiltonian describing finite electron bands for both donor and acceptor, in the continuous limit, has the form $\mathcal{H}(t) = \mathcal{H}_0(t) + W$, where

$$\mathcal{H}_0(t) = \int (E_d + \lambda_d(t))|E_d\rangle\langle E_d|\varrho(E_d)dE_d + \int (E_a + \lambda_a(t))|E_a\rangle\langle E_a|\varrho(E_a)dE_a,$$

$$W = \int\int dE_d dE_a \varrho(E_d)\varrho(E_a)\Big(V(E_d, E_a)|E_d\rangle\langle E_a| + \text{h.c.}\Big), \tag{101}$$

$\varrho(E_d)$ and $\varrho(E_a)$ are the densities of electron states of the donor and acceptor, respectively. We assume that the amplitude of transition is a smoothly varying function of energy, so that one can approximate, $V(E_d, E_a) \approx V = \text{const}$. In the interaction representation, $\tilde{\rho} = e^{i\int_0^t \mathcal{H}_0(\tau)d\tau}\rho e^{-i\int_0^t \mathcal{H}_0(\tau)d\tau}$, the equations of motion are given by

$$\frac{d}{dt}\tilde{\rho}_{E_dE'_d} = i\int dE_a\varrho(E_a)\big(\tilde{\rho}_{E_dE_a}\tilde{V}^*(E_a, E'_d, t) - \tilde{V}(E_d, E_a, t)\tilde{\rho}_{E_aE'_d}\big), \tag{102}$$

$$\frac{d}{dt}\tilde{\rho}_{E_aE'_a} = i\int dE''_d\varrho(E''_d)\big(\tilde{\rho}_{E_aE''_d}\tilde{V}^*(E''_d, E'_a, t) - \tilde{V}(E_a, E''_d, t)\tilde{\rho}_{E''_dE'_a}\big), \tag{103}$$

$$\frac{d}{dt}\tilde{\rho}_{E_dE_a} = i\int dE''_d\varrho(E''_d)\tilde{\rho}_{E_dE''_d}\tilde{V}(E''_d, E_a, t) - i\int dE''_a\varrho(E''_a)\tilde{V}(E_d, E''_a, t)\tilde{\rho}_{E''_aE_a}, \tag{104}$$

$$\frac{d}{dt}\tilde{\rho}_{E_aE_d} = i\int dE''_a\varrho(E''_a)\tilde{\rho}_{E_aE''_a}\tilde{V}^*(E''_a, E_d, t) - i\int dE''_d\varrho(E''_d)\tilde{V}^*(E_a, E''_d, t)\tilde{\rho}_{E''_dE_d}, \tag{105}$$

where $\tilde{V}(E_d, E_a, t) = Ve^{i(E_d - E_a)t}e^{i\varphi_{da}(t)}$ and $\varphi_{da}(t) = (g_d - g_a)\int_0^t \xi(t')dt'$.

We assume that initially, $\tilde{\rho}_{E_dE_a}(0) = \tilde{\rho}_{E_dE_a}(0) = 0$, and employ the same procedure as in the previous section. After splitting of correlations, we obtain

$$
\begin{aligned}
&\frac{d}{dt}\langle\tilde{\rho}_{E_d,E'_d}(t)\rangle = \\
&-\int_0^t dt' \iint dE_a\varrho(E_a)dE''_d\varrho(E''_d)\Big(\langle\tilde{\rho}_{E_dE''_d}(t')\rangle\langle\tilde{V}(E''_d,E_a,t')\tilde{V}^*(E_a,E'_d,t)\rangle + \langle\tilde{V}(E_d,E_a,t)\tilde{V}^*(E_a,E''_d,t')\rangle\langle\tilde{\rho}_{E''_dE'_d}(t')\rangle\Big) \\
&+\int_0^t dt' \iint dE_a\varrho(E_a)dE''_a\varrho(E''_a)\Big(\langle\tilde{\rho}_{E''_aE_a}(t')\rangle\langle\tilde{V}(E_d,E''_a,t')\tilde{V}^*(E_a,E'_d,t)\rangle + \langle\tilde{V}(E_d,E_a,t)\tilde{V}^*(E''_a,E'_d,t')\rangle\langle\tilde{\rho}_{E_aE''_a}(t')\rangle\Big),
\end{aligned} \tag{106}
$$

$$
\begin{aligned}
&\frac{d}{dt}\langle\tilde{\rho}_{E_aE'_a}(t)\rangle = \\
&\int_0^t dt' \iint dE'_d\varrho(E'_d)dE''_d\varrho(E''_d)\Big(\langle\tilde{\rho}_{E'_dE''_d}(t')\rangle\langle\tilde{V}^*(E''_d,E'_a,t)\tilde{V}(E_a,E'_d,t')\rangle + \langle\tilde{V}(E'_d,E'_a,t)\tilde{V}^*(E_a,E''_d,t')\rangle\langle\tilde{\rho}_{E''_dE'_d}(t')\rangle\Big) \\
&-\int_0^t dt' \iint dE'_d\varrho(E'_d)dE''_a\varrho(E''_a)\Big(\langle\tilde{\rho}_{E'_aE''_a}(t')\rangle\langle\tilde{V}^*(E''_d,E'_a,t)\tilde{V}(E_a,E''_d,t')\rangle + \langle\tilde{V}(E''_d,E''_a,t')\tilde{V}^*(E_a,E''_d,t)\rangle\langle\tilde{\rho}_{E''_aE'_a}(t')\rangle\Big).
\end{aligned} \tag{107}
$$

These equations can be recast as,

$$
\begin{aligned}
&\frac{d}{dt}\langle\tilde{\rho}_{E_d,E'_d}(t)\rangle = \\
&-|V|^2\int_0^t dt'\Phi(t,t') \iint dE_a\varrho(E_a)dE''_d\varrho(E''_d)\Big(\langle\tilde{\rho}_{E_dE''_d}(t')\rangle e^{-i(E_a-E'_d)t}e^{i(E''_d-E_a)t'} + e^{i(E_d-E_a)t}e^{-i(E_a-E''_d)t'}\langle\tilde{\rho}_{E''_dE'_d}(t')\rangle\Big) \\
&+|V|^2\int_0^t dt\Phi(t,t')' \iint dE_a\varrho(E_a)dE''_a\varrho(E''_a)\Big(\langle\tilde{\rho}_{E''_aE_a}(t')\rangle e^{-i(E_a-E'_d)t}e^{i(E_d-E''_a)t'} + e^{i(E_d-E_a)t}e^{-i(E''_a-E'_d)t'}\langle\tilde{\rho}_{E_aE''_a}(t')\rangle\Big),
\end{aligned} \tag{108}
$$

$$
\begin{aligned}
&\frac{d}{dt}\langle\tilde{\rho}_{E_aE'_a}(t)\rangle = \\
&|V|^2\int_0^t dt'\Phi(t,t') \iint dE'_d\varrho(E'_d)dE''_d\varrho(E''_d)\Big(\langle\tilde{\rho}_{E'_dE''_d}(t')\rangle e^{-i(E''_d-E'_a)t}e^{i(E_a-E'_d)t'} + e^{i(E'_d-E'_a)t}e^{-i(E_a-E''_d)t'}\langle\tilde{\rho}_{E''_dE'_d}(t')\rangle\Big) \\
&-|V|^2\int_0^t dt'\Phi(t,t') \iint dE'_d\varrho(E'_d)dE''_a\varrho(E''_a)\Big(\langle\tilde{\rho}_{E'_aE''_a}(t')\rangle e^{-i(E''_d-E'_a)t}e^{i(E_a-E''_d)t'} + e^{-i(E_a-E''_d)t}e^{i(E''_d-E''_a)t'}\langle\tilde{\rho}_{E''_aE'_a}(t')\rangle\Big),
\end{aligned} \tag{109}
$$

where $\Phi(t,t') = \langle e^{i\varphi_{da}(t)}e^{-i\varphi_{da}(t')}\rangle$.

Further, we consider a Gaussian density of states in the donor and acceptor bands centered at some energies, $E_0^{(a)}$, for the acceptor, and, $E_0^{(d)}$, for the donor,

$$
\varrho(E_d) = \varrho_d e^{-\alpha_1(E_d-E_0^{(d)})^2}, \tag{110}
$$

$$
\varrho(E_a) = \varrho_a e^{-\alpha_2(E_a-E_0^{(a)})^2}. \tag{111}
$$

We denote by $\langle\tilde{\rho}_{11}(t)\rangle = \int\langle\tilde{\rho}_{E_dE_d}\rangle(t)\varrho(E_d)dE_d$ and $\langle\tilde{\rho}_{22}(t)\rangle = \int\langle\tilde{\rho}_{E_aE_a}\rangle(t)\varrho(E_a)dE_a$ the probabilities to populate the donor and the acceptor, respectively. A computation yields

$$\langle\tilde{\rho}_{11}(t)\rangle = \int \langle\tilde{\rho}_{E_dE_d}\rangle(t)\varrho(E_d)dE_d \approx \langle\tilde{\rho}_{E_0^{(d)}E_0^{(d)}}(t')\rangle\varrho_0\sqrt{\frac{\pi}{\alpha}}, \tag{112}$$

$$\langle\tilde{\rho}_{22}(t)\rangle = \int \langle\tilde{\rho}_{E_aE_a}\rangle(t)\varrho(E_a)dE_a \approx \langle\tilde{\rho}_{E_0^{(a)}E_0^{(a)}}(t')\rangle\varrho_0\sqrt{\frac{\pi}{\alpha}}, \tag{113}$$

$$\begin{aligned}&\int dE_a\varrho(E_a)\iint dE_d dE'_d\varrho(E_d)\varrho(E'_d)e^{i(E_a-E_d)t}e^{i(E'_d-E_a)t'}\big\langle\tilde{\rho}_{E_dE'_d}(t')\big\rangle\\ &\approx\langle\tilde{\rho}_{11}(t)\rangle\exp\Big(i\varepsilon(t-t')-\frac{(t-t')^2}{4\alpha_2}-\frac{t^2+t'^2}{4\alpha_1}\Big)\end{aligned} \tag{114}$$

$$\begin{aligned}&\int dE_d\varrho(E_d)\iint dE_a dE'_a\varrho(E_a)\varrho(E'_a)e^{i(E_d-E_a)t}e^{i(E'_a-E_d)t'}\big\langle\tilde{\rho}_{E_aE'_a}(t')\big\rangle\\ &\approx\langle\tilde{\rho}_{22}(t)\rangle\exp\Big(i\varepsilon(t-t')-\frac{(t-t')^2}{4\alpha_1}-\frac{t^2+t'^2}{4\alpha_2}\Big),\end{aligned} \tag{115}$$

where $\varepsilon = E_0^{(d)} - E_0^{(a)}$. Next, employing Eqs. (112) – (115), we obtain from Eqs. (108) – (109) the following system of integro-differential equations for the diagonal components of the density matrix,

$$\frac{d}{dt}\langle\tilde{\rho}_{11}(t)\rangle = -\int_0^t K_1(t,t')\langle\tilde{\rho}_{11}(t')\rangle dt' + \int_0^t K_2(t,t')\big(\langle\tilde{\rho}_{22}(t')\rangle dt', \tag{116}$$

$$\frac{d}{dt}\langle\tilde{\rho}_{22}(t)\rangle = \int_0^t K_1(t,t')\langle\tilde{\rho}_{11}(t')\rangle dt' - \int_0^t K_2(t,t')\langle\tilde{\rho}_{22}(t')\rangle dt', \tag{117}$$

where

$$K_1(t,t') = 2v^2\Phi(t,t')\cos(\varepsilon(t-t'))\exp\Big(-\frac{(t-t')^2}{4\alpha_2}-\frac{t^2+t'^2}{4\alpha_1}\Big), \tag{118}$$

$$K_2(t,t') = 2v^2\Phi(t,t')\cos(\varepsilon(t-t'))\exp\Big(-\frac{(t-t')^2}{4\alpha_1}-\frac{t^2+t'^2}{4\alpha_2}\Big). \tag{119}$$

For $|\int_0^t(t-t')K_{1,2}(t,t')dt'| \ll 1$, the system of integro-differential equations (116) and (117) can be approximated by the system of ordinary differential equations,

$$\frac{d}{dt}\langle\rho_{11}\rangle = -\mathfrak{R}_1(t)\langle\rho_{11}\rangle + \mathfrak{R}_2(t)\langle\rho_{22}\rangle, \tag{120}$$

$$\frac{d}{dt}\langle\rho_{22}\rangle = \mathfrak{R}_1(t)\langle\rho_{11}\rangle - \mathfrak{R}_2(t)\langle\rho_{22}\rangle, \tag{121}$$

where $\mathfrak{R}_{1,2}(t) = \int_0^t K_{1,2}(t,t')dt'$. A computation yields

$$\mathfrak{R}_1(t) = \frac{\sqrt{\pi}\,v^2}{p}\Bigg\{\exp\Big(-\frac{t^2}{2\alpha_1}+\frac{(t/2\alpha_1-i\varepsilon)^2}{p^2}\Big)\Big(\mathrm{erf}\Big(\frac{pt}{2}-\frac{(t/2\alpha_1-i\varepsilon)}{p}\Big)+\mathrm{erf}\Big(\frac{(t/2\alpha_1-i\varepsilon)}{p}\Big)\Big)+\text{c.c.}\Bigg\}, \tag{122}$$

$$\mathfrak{R}_2(t) = \frac{\sqrt{\pi}\,v^2}{p}\Bigg\{\exp\Big(-\frac{t^2}{2\alpha_2}+\frac{(t/2\alpha_2-i\varepsilon)^2}{p^2}\Big)\Big(\mathrm{erf}\Big(\frac{pt}{2}-\frac{(t/2\alpha_2-i\varepsilon)}{p}\Big)+\mathrm{erf}\Big(\frac{(t/2\alpha_2-i\varepsilon)}{p}\Big)\Big)+\text{c.c.}\Bigg\}, \tag{123}$$

where $p = \sqrt{1/\alpha_1 + 1/\alpha_2 + 2(D\sigma)^2}$.

Following the same procedure as in the Section I, one can show that the condition for validity of approximating the integro-differential equations by the ordinary differential equations is modified as follows:

$$|D\sqrt{\chi(0)}| \ll \frac{d}{dt}\ln\chi(t)\Big|_{t=0} \ll \frac{3p^2}{2\varepsilon}, \tag{124}$$

$$\Big|\Big(\frac{\partial\tilde{\mathfrak{R}}_{1,2}(t)}{\partial\varepsilon}-\frac{\partial\tilde{\mathfrak{R}}^*_{1,2}(t)}{\partial\varepsilon}\Big)\Big| \ll 1, \quad \text{and} \quad Z(t) = \frac{|\Delta\rho(t)|}{\langle\rho_{22}(t)\rangle} \ll 1, \tag{125}$$

where

$$\tilde{\mathfrak{R}}_1(t) = \frac{\sqrt{\pi}\, v^2}{p} \exp\Big(-\frac{t^2}{2\alpha_1} + \frac{(t/2\alpha_1 - i\varepsilon)^2}{p^2}\Big)\Big(\text{erf}\Big(\frac{pt}{2} - \frac{(t/2\alpha_1 - i\varepsilon)}{p}\Big) + \text{erf}\Big(\frac{(t/2\alpha_1 - i\varepsilon)}{p}\Big)\Big), \tag{126}$$

$$\tilde{\mathfrak{R}}_2(t) = \frac{\sqrt{\pi}\, v^2}{p} \exp\Big(-\frac{t^2}{2\alpha_2} + \frac{(t/2\alpha_2 - i\varepsilon)^2}{p^2}\Big)\Big(\text{erf}\Big(\frac{pt}{2} - \frac{(t/2\alpha_2 - i\varepsilon)}{p}\Big) + \text{erf}\Big(\frac{(t/2\alpha_2 - i\varepsilon)}{p}\Big)\Big), \tag{127}$$

$$|\Delta\rho(t)| = \Big|\text{Re}\int_0^t dt'\Big(\frac{\partial\tilde{\mathfrak{R}}_1(t')}{\partial\varepsilon}\langle\tilde{X}_1^*(t')\rangle + it'\tilde{\mathfrak{R}}_1(t')\big(\langle\tilde{X}_1(t')\rangle - \langle\tilde{X}_1^*(t')\rangle\big)\Big) + \text{Re}\int_0^t dt'\Big(\frac{\partial\tilde{\mathfrak{R}}_2(t')}{\partial\varepsilon}\langle\tilde{X}_2^*(t')\rangle + it'\tilde{\mathfrak{R}}_2(t')\big(\langle\tilde{X}_2(t')\rangle - \langle\tilde{X}_2^*(\tau')\rangle\big)\Big)\Big|. \tag{128}$$

Here we denote,

$$\langle\tilde{X}_1(t)\rangle = \varrho_0\sqrt{\frac{\pi}{\alpha_1}}\frac{\partial}{\partial E_d'}\langle\tilde{\rho}_{E_dE_d'}(t)\rangle\Big|_{E_d=E_d'=E_0^{(a)}} \quad \text{and} \quad \langle\tilde{X}_2(t)\rangle = \varrho_0\sqrt{\frac{\pi}{\alpha_2}}\frac{\partial}{\partial E_a'}\langle\tilde{\rho}_{E_aE_a'}(t)\rangle\Big|_{E_a=E_a'=E_0^{(a)}}. \tag{129}$$

To proceed further, it is convenient to define a scaled time, $\tau = pt$, and the dimensionless complex dynamical rates, $\tilde{\mathfrak{r}}_{1,2}(\tau) = (1/p)\mathfrak{R}_{1,2}(\tau)$:

$$\tilde{\mathfrak{r}}_1(\tau) = \sqrt{\pi}a^2 \exp\Big(-\frac{\mu_1^2\tau^2}{2} + \Big(\frac{\mu_1\tau}{2} - i\eta\Big)^2\Big)\Big(\text{erf}\Big(\frac{\tau}{2} - \frac{\mu_1\tau}{2} + i\eta\Big) + \text{erf}\Big(\frac{\mu_1\tau}{2} - i\eta\Big)\Big), \tag{130}$$

$$\tilde{\mathfrak{r}}_2(\tau) = \sqrt{\pi}a^2 \exp\Big(-\frac{\mu_2^2\tau^2}{2} + \Big(\frac{\mu_2\tau}{2} - i\eta\Big)^2\Big)\Big(\text{erf}\Big(\frac{\tau}{2} - \frac{\mu_2\tau}{2} + i\eta\Big) + \text{erf}\Big(\frac{\mu_2\tau}{2} - i\eta\Big)\Big), \tag{131}$$

where, $a = v/p$, $\eta = \varepsilon/p$ and $\mu_{1,2} = 1/\sqrt{\alpha_{1,2}}p$. We obtain

$$|\Delta\rho(\tau)| = \Big|\mu_1^2\text{Re}\int_0^\tau d\tau'\Big(\frac{\partial\tilde{\mathfrak{r}}_1(\tau')}{\partial\eta}\langle\tilde{X}_1^*(\tau')\rangle + i\tau'\tilde{\mathfrak{r}}_1(\tau')\big(\langle\tilde{X}_1(\tau')\rangle - \langle\tilde{X}_1^*(\tau')\rangle\big)\Big) + \mu_2^2\text{Re}\int_0^\tau d\tau'\Big(\frac{\partial\tilde{\mathfrak{r}}_2(\tau')}{\partial\eta}\langle\tilde{X}_2^*(\tau')\rangle + i\tau'\tilde{\mathfrak{r}}_2(\tau')\big(\langle\tilde{X}_2(\tau')\rangle - \langle\tilde{X}_2^*(\tau')\rangle\big)\Big)\Big|. \tag{132}$$

The functions, $\langle\rho_{11}(\tau)\rangle$, $\langle\rho_{22}(\tau)\rangle$ and $\langle\tilde{X}_{1,2}(\tau)\rangle$, obey the the following differential equations

$$\frac{d}{d\tau}\langle\rho_{11}(\tau)\rangle = -\mathfrak{r}_1(\tau)\big\langle\rho_{11}(\tau)\big\rangle + \mathfrak{r}_2(\tau)\big\langle\rho_{22}(\tau)\big\rangle, \tag{133}$$

$$\frac{d}{d\tau}\langle\rho_{22}(\tau)\rangle = \mathfrak{r}_1(\tau)\big\langle\rho_{11}(\tau)\big\rangle - \mathfrak{r}_2(\tau)\big\langle\rho_{22}(\tau)\big\rangle, \tag{134}$$

$$\frac{d}{dt}\langle\tilde{X}_1(\tau)\rangle = -i\tau\mathfrak{r}_1(\tau)\big\langle\rho_{11}(\tau)\big\rangle - \frac{\partial\tilde{\mathfrak{r}}_1^*(\tau)}{\partial\eta}\big\langle\rho_{11}(\tau)\big\rangle + i\tau\tilde{\mathfrak{r}}_2(\tau)\big\langle\rho_{22}(\tau)\big\rangle, \tag{135}$$

$$\frac{d}{dt}\langle\tilde{X}_2(\tau)\rangle = -i\tau\mathfrak{r}_2(\tau)\big\langle\rho_{22}(\tau)\big\rangle - \frac{\partial\tilde{\mathfrak{r}}_2^*(\tau)}{\partial\eta}\big\langle\rho_{22}(\tau)\big\rangle + i\tau\tilde{\mathfrak{r}}_1(\tau)\big\langle\rho_{11}(\tau)\big\rangle. \tag{136}$$

Putting all results together, we conclude that in the time interval, $0 < t < \infty$, the original system of integro-differential equations (112) and (115) can be approximated by the system of ordinary differential equations (120) and (121), if the following conditions hold:

$$|D\sqrt{\chi(0)}| \ll \frac{d}{dt}\ln\chi(t)\Big|_{t=0} \ll \frac{3p^2}{2\varepsilon}, \tag{137}$$

$$W(\tau, a, \mu, \eta) = \Big|\Big(\frac{\partial\tilde{\mathfrak{r}}_{1,2}(\tau)}{\partial\eta} - \frac{\partial\tilde{\mathfrak{r}}_{1,2}^*(\tau)}{\partial\eta}\Big)\Big| \ll 1, \tag{138}$$

$$Z(\tau, a, \mu, \eta) = \frac{|\Delta\rho(\tau)|}{\langle\rho_{22}(\tau)\rangle} \ll 1. \tag{139}$$

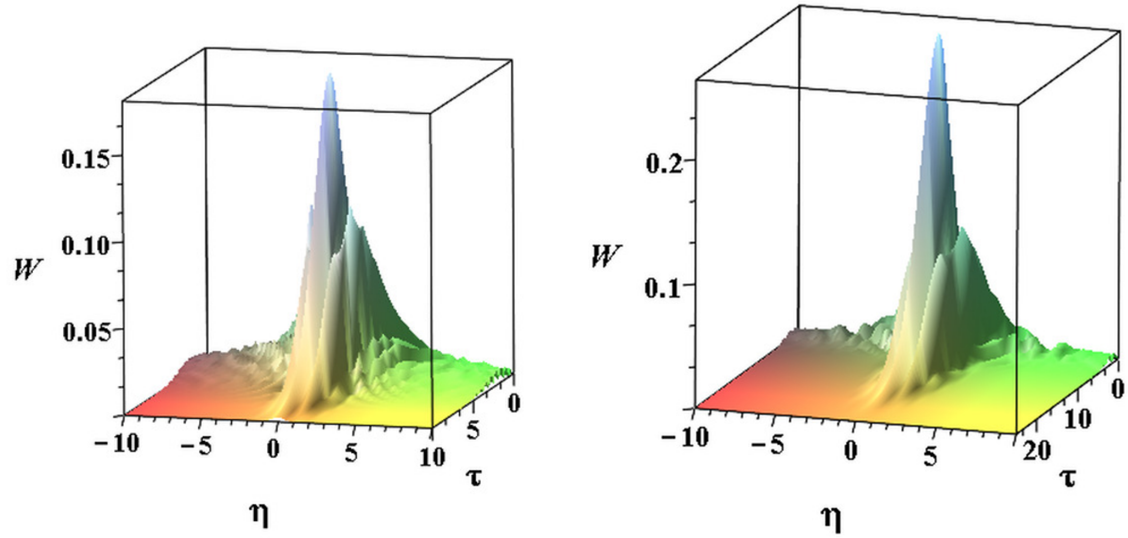

FIG. 2: (Color online) Plot of the function W. Dependence of W on the scaled time, τ, and parameter, $\eta = \varepsilon/p$. Left: $\mu = 0.5$, $v/p = 0.2$. Right: $\mu = 0.25$, $v/p = 0.2$.

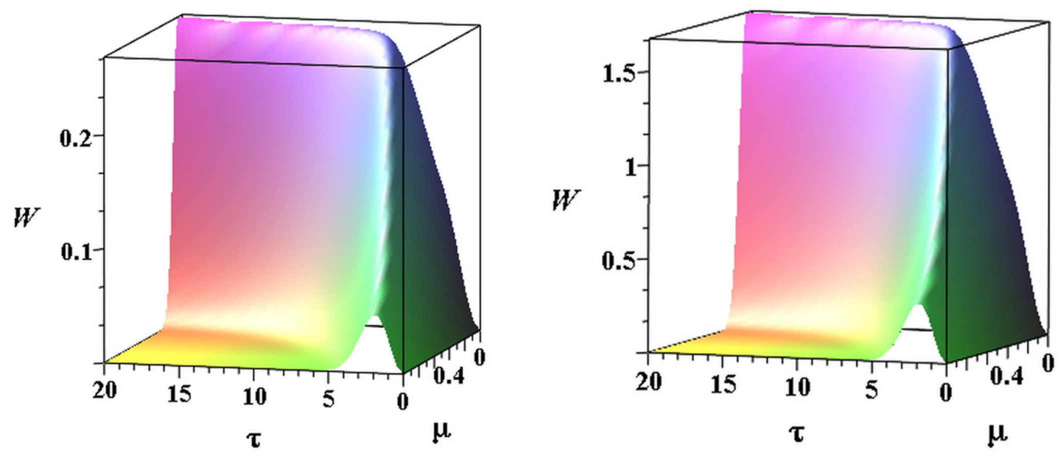

FIG. 3: (Color online) Plot of the function W. Dependence W on the scaled time τ and parameter μ ($\eta = 0$). Left: $v/p = 0.2$. Right: $v/p = 0.5$.

In Figs. 2 and 3, the function, W, is presented. As one can see, for the chosen values of parameters, the condition (138) is satisfied. As it follows from analysis of Fig. 3, and taking into account that in our perturbative theory the "small" parameter is, v/p, the condition (138) should be replaced by a more strong inequality: $v/p \ll 1$. Thus, the conditions of validity of approximation leading to the rate-type equations (120) and (121) can be written in its final form as,

$$|D\sqrt{\chi(0)}| \ll \frac{d}{dt}\ln\chi(t)\Big|_{t=0} \ll \frac{3p^2}{2\varepsilon}, \tag{140}$$

$$\frac{v}{p} \ll 1, \quad \text{and} \quad \frac{|\Delta\rho(t)|}{\langle\rho_{22}(t)\rangle} \ll 1. \tag{141}$$

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Appendix A: Correlation splitting

In this section, we consider the correlation splitting for the random telegraph process (RTP) $\xi(t)$ having the property $\langle\xi(t)\rangle = 0$ and the correlation function

$$\chi(t-t') = \langle\xi(t)\xi(t')\rangle = \sigma^2 e^{-2\gamma(t-t')}, \quad t \geq t'. \tag{A1}$$

Thus $\chi(0) = \sigma^2$. We also have $\xi(t)^2 = \sigma^2$. Let $M_n(t_1, t_2, \ldots, t_n) = \langle\xi(t_1)\ldots\xi(t_n)\rangle$ for ordered times $t_1 \geq t_2 \geq \cdots \geq t_n$. Then we have (see [1, 2])

$$M_n(t_1, t_2, \ldots, t_n) = \langle\xi(t_1)\xi(t_2)\rangle M_{n-2}(t_3, \ldots, t_n). \tag{A2}$$

The RTP is conveniently described by its characteristic functional, defined by [2]

$$\Phi[t; \nu(\tau)] = \Big\langle \exp\Big\{ i \int_0^t d\tau \xi(\tau)\nu(\tau)\Big\}\Big\rangle, \tag{A3}$$

where, $\nu(\tau)$ is arbitrary (sufficiently smooth) function. Applying Eq. (A2) and using the Taylor expansion of Eq. (A3), we obtain an exact integral equation for the characteristic functional, $\Phi[t; \nu(\tau)]$,

$$\Phi[t; \nu(\tau)] = 1 - \int_0^t dt_1 \int_0^{t_1} dt_2 \chi(t_1 - t_2)\nu(t_1)\nu(t_2)\Phi[t_2; \nu(\tau)]. \tag{A4}$$

One can transform this integral equation into the integro-differential equation,

$$\frac{d}{dt}\Phi[t; \nu(\tau)] = -\nu(t)\int_0^t dt_1 \chi(t - t_1)\nu(t_1)\Phi[t_1; \nu(\tau)]. \tag{A5}$$

Let $R[t; \xi(\tau)]$ be an arbitrary functional. Then, for correlators, $\langle\xi(t)R[t; \xi(\tau)]\rangle$ and $\langle\xi(t_1)\xi(t_2)R[t; \xi(\tau)]\rangle$, one can show that the following correlation splitting formulas hold [2]:

$$\langle\xi(t)R[t; \xi(\tau)]\rangle = \int_0^t dt_1 \chi(t - t_1)\Big\langle \frac{\delta}{\delta\xi(t_1)}\tilde{R}[t, t_1; \xi(\tau)]\Big\rangle, \tag{A6}$$

$$\langle\xi(t_1)\xi(t_2)R[t; \xi(\tau)]\rangle = \chi(t_1 - t_2)\langle R[t; \xi(\tau)]\rangle,\ t_1 \geq t_2 \geq \tau, \tag{A7}$$

where $\tilde{R}[t, t_1; \xi(\tau)] = R[t; \xi(\tau)]\theta(t_1 - \tau + 0)$, and $\theta(z)$ is the Heaviside step function. The differentiation of (A6) yields the differential formula [1, 2],

$$\Big(\frac{d}{dt} + 2\gamma\Big)\langle\xi(t)R[t; \xi(\tau)]\rangle = \Big\langle \xi(t)\frac{d}{dt}R[t; \xi(\tau)]\Big\rangle. \tag{A8}$$

Theorem 1.1 *Let $R[t;\xi(\tau)]$ be an arbitrary functional. Then, if $\xi(t)$ is the RTP the following correlation splitting holds:*

$$\Big\langle \exp\Big\{i\int_0^t d\tau\xi(\tau)\nu(\tau)\Big\}R[t';\xi(\tau)]\Big\rangle = \Phi[t;\nu(\tau)]\big\langle R[t';\xi(\tau)]\big\rangle + \Psi[t,t';\nu(\tau)]\big\langle R^{\xi}[t';\xi(\tau)]\big\rangle, \quad t\ge t', \tag{A9}$$

where $\langle R^{\xi}[t';\xi(\tau)]\rangle = \langle \xi(t')R[t';\xi(\tau)]\rangle$ *and*

$$\Psi[t,t';\nu(\tau)] = \frac{\chi(t-t')}{i\nu(t)\chi^2(0)}\frac{d}{dt}\Phi[t;\nu(\tau)]. \tag{A10}$$

Proof. Expanding the functional, $\exp\Big\{i\int_0^t d\tau\xi(\tau)\nu(\tau)\Big\}$, in the Taylor series, we obtain

$$\Big\langle \exp\Big\{i\int_0^t d\tau\xi(\tau)\nu(\tau)\Big\}R[t';\xi(\tau)]\Big\rangle = \Big\langle\Big(\sum_{n=0}^{\infty}\frac{i^{2n}}{2n!}Z_{2n}(t) + \sum_{n=0}^{\infty}\frac{i^{2n+1}}{(2n+1)!}Z_{2n+1}(t)\Big)R[t';\xi(\tau)]\Big\rangle, \tag{A11}$$

where $Z_n(t) = \prod_{k=1}^{n}\int_0^t \nu(t_k)\xi(t_k)dt_k$, $(t_1\ge t_2\ge\cdots\ge t_n)$. Using the property of the RTP, $\xi^2(t') = \chi(0)$, and the recurrence relationship (A2), one can recast (A11) as,

$$\begin{aligned}\Big\langle \exp\Big\{i\int_0^t d\tau\xi(\tau)\nu(\tau)\Big\}R[t';\xi(\tau)]\Big\rangle &= \Big(\sum_{n=0}^{\infty}\frac{i^{2n}}{2n!}\Big\langle Z_{2n}(t)\Big\rangle\Big)\Big\langle R[t';\xi(\tau)]\Big\rangle \\ &+ \frac{1}{\chi(0)}\Big(\sum_{n=0}^{\infty}\frac{i^{2n+1}}{(2n+1)!}\Big\langle Z_{2n+1}(t)\xi(t')\Big\rangle\Big)\Big\langle R^{\xi}[t';\xi(\tau)]\Big\rangle.\end{aligned} \tag{A12}$$

Next, expanding $\Phi[t;\nu(\tau)]$ in the Taylor series and taking into account that $\langle Z_{2n+1}(t)\rangle = 0$, we obtain

$$\Phi[t;\nu(\tau)] = \sum_{n=0}^{\infty}\frac{i^n}{n!}\Big\langle Z_n(t)\Big\rangle = \sum_{n=0}^{\infty}\frac{i^{2n}}{2n!}\Big\langle Z_{2n}(t)\Big\rangle. \tag{A13}$$

A similar consideration yields

$$\Big\langle \exp\Big\{i\int_0^t d\tau\xi(\tau)\nu(\tau)\Big\}\xi(t')\Big\rangle = \sum_{n=0}^{\infty}\frac{i^n}{n!}\Big\langle Z_n(t)\xi(t')\Big\rangle = \sum_{n=0}^{\infty}\frac{i^{2n+1}}{(2n+1)!}\Big\langle Z_{2n+1}(t)\xi(t')\Big\rangle. \tag{A14}$$

Using Eqs. (A13) and (A14), one can rewrite (A12) as,

$$\Big\langle \exp\Big\{i\int_0^t d\tau\xi(\tau)\nu(\tau)\Big\}R[t';\xi(\tau)]\Big\rangle = \Phi[t;\nu(\tau)]\Big\langle R[t';\xi(\tau)]\Big\rangle + \frac{1}{\chi(0)}\Big\langle \exp\Big\{i\int_0^t d\tau\xi(\tau)\nu(\tau)\Big\}\xi(t')\Big\rangle\Big\langle R^{\xi}[t';\xi(\tau)].\Big\rangle \tag{A15}$$

Once again using the relationship $\xi^2(t) = \chi(0)$ and Eq. (A2), we obtain

$$\sum_{n=0}^{\infty}\frac{i^n}{n!}\Big\langle Z_n(t)\xi(t')\Big\rangle = \frac{1}{\chi(0)}\sum_{n=0}^{\infty}\frac{i^n}{n!}\Big\langle \xi(t)Z_n(t)\xi(t)\xi(t')\Big\rangle = \frac{1}{\chi(0)}\sum_{n=0}^{\infty}\frac{i^{2n+1}}{(2n+1)!}\Big\langle \xi(t)Z_{2n+1}(t)\Big\rangle\langle\xi(t)\xi(t')\rangle. \tag{A16}$$

The last term can be rewritten as,

$$\frac{1}{\chi(0)}\sum_{n=0}^{\infty}\frac{i^{2n+1}}{(2n+1)!}\Big\langle \xi(t)Z_{2n+1}(t)\Big\rangle\langle\xi(t)\xi(t')\rangle = \frac{\chi(t-t')}{\nu(t)\chi(0)}\sum_{n=0}^{\infty}\frac{i^n}{n!}\frac{d}{dt}\Big\langle Z_n(t)\Big\rangle = \frac{\chi(t-t')}{i\nu(t)\chi(0)}\frac{d}{dt}\Phi[t;\nu(\tau)]. \tag{A17}$$

From here it follows,

$$\Big\langle \exp\Big\{i\int\limits_0^t d\tau\xi(\tau)\nu(\tau)\Big\}\xi(t')\Big\rangle = \frac{\chi(t-t')}{i\nu(t)\chi(0)}\Big\langle \exp\Big\{i\int\limits_0^t d\tau\xi(\tau)\nu(\tau)\Big\}\xi(t')\Big\rangle = \frac{\chi(t-t')}{i\nu(t)\chi(0)}\frac{d}{dt}\Phi[t;\nu(\tau)]. \tag{A18}$$

Inserting this result into the r.h.s. of Eq. (A12), we obtain

$$\Big\langle \exp\Big\{i\int\limits_0^t d\tau\xi(\tau)\nu(\tau)\Big\}R[t';\xi(\tau)]\Big\rangle = \Phi[t;\nu(\tau)]\big\langle, R[t';\xi(\tau)]\big\rangle + \Psi[t,t';\nu(\tau)]\big\langle R^{\xi}[t';\xi(\tau)]\big\rangle, \quad t\ge t', \tag{A19}$$

where

$$\Psi[t,t';\nu(\tau)] = \frac{\chi(t-t')}{i\nu(t)\chi^2(0)}\frac{d}{dt}\Phi[t;\nu(\tau)]. \tag{A20}$$

□

Theorem 1.2 *Let $R[t;\xi(\tau)]$ be an arbitrary functional. Then, if $\xi(t)$ is the RTP, the following correlation splitting holds:*

$$\Big\langle \exp\Big\{i\int\limits_0^{t-t'} d\tau\xi(\tau)\nu(\tau)\Big\}R[t';\xi(\tau)]\Big\rangle = \Phi[t-t';\nu(\tau)]\big\langle R[t';\xi(\tau)]\big\rangle - \frac{1}{i\nu(t)\chi(0)}\frac{d}{dt'}\Phi[t-t';\nu(\tau)]\big\langle R^{\xi}[t';\xi(\tau)]\big\rangle, \quad t\ge t', \tag{A21}$$

where $\big\langle R^{\xi}[t';\xi(\tau)]\big\rangle = \big\langle \xi(t')R[t';\xi(\tau)]\big\rangle$ *and* $\Phi[t-t';\nu(\tau)] = \Big\langle \exp\Big\{i\int\limits_0^{t-t'} d\tau\xi(\tau)\nu(\tau)\Big\}\Big\rangle$.

Proof. Since for the stationary random process, $\xi(t)=\xi(t-t')$, one can recast the functional $\Phi[t-t';\nu(\tau)]$ as follows: $\Phi[t-t';\nu(\tau)] = \langle e^{i\varphi(t)}e^{-i\varphi(t')}\rangle$, where $\varphi(t)=\int_0^t d\tau\xi(\tau)\nu(\tau)$. Next, expanding $e^{i\varphi(t)}$ and $e^{-i\varphi(t')}\rangle$ in the Taylor series, one can write

$$\Big\langle \exp\Big\{i\int\limits_0^{t-t'} d\tau\xi(\tau)\nu(\tau)\Big\}R[t';\xi(\tau)]\Big\rangle = \Big\langle\Big(\sum_{n=0}^{\infty}\frac{i^n}{n!}Z_n(t)\Big)\Big(\sum_{m=0}^{\infty}\frac{(-i)^m}{m!}Z_m(t')\Big)R[t';\xi(\tau)]\Big\rangle, \tag{A22}$$

where $Z_n(t)=\prod_{k=1}^n\int_0^t \nu(t_k)\xi(t_k)dt_k$ and $Z_m(t)=\prod_{k=1}^m\int_0^t \nu(\tau_k)\xi(\tau_k)d\tau_k$, $(t_1\ge t_2\ge\cdots\ge t_n\ge\tau_1\ge\tau_2\ge\cdots\ge\tau_m)$.
Using the results of Theorem 1.1, after some algebra we obtain,

$$\begin{aligned}&\Big\langle \exp\Big\{i\int\limits_0^{t-t'} d\tau\xi(\tau)\nu(\tau)\Big\}R[t';\xi(\tau)]\Big\rangle = \Phi[t-t';\nu(\tau)]\big\langle R[t';\xi(\tau)]\big\rangle\\ &+\Big\langle\Big(\sum_{m,n=0}^{\infty}\frac{i^{2n}}{2n!}\frac{(-i)^{2m+1}}{(2m+1)!}Z_{2n}(t)Z_{2m+1}(t') + \sum_{m,n=0}^{\infty}\frac{i^{2n+1}}{(2n+1)!}\frac{(-i)^{2m}}{2m!}Z_{2n+1}(t)Z_{2m}(t')\Big)R[t';\xi(\tau)]\Big\rangle.\end{aligned} \tag{A23}$$

Further, employing the relationship $\xi^2(t')=\chi(0)$, one can recast the r.h.s. of Eq. (A23) as,

$$\begin{aligned}&\Big\langle\Big(\sum_{m,n=0}^{\infty}\frac{i^{2n}}{2n!}\frac{(-i)^{2m+1}}{(2m+1)!}Z_{2n}(t)Z_{2m+1}(t') + \sum_{m,n=0}^{\infty}\frac{i^{2n+1}}{(2n+1)!}\frac{(-i)^{2m}}{2m!}Z_{2n+1}(t)Z_{2m}(t')\Big)R[t';\xi(\tau)]\Big\rangle\\ &=\frac{1}{\chi(0)}\Big(\Big\langle\sum_{m,n=0}^{\infty}\frac{i^{2n}}{2n!}\frac{(-i)^{2m+1}}{(2m+1)!}Z_{2n}(t)Z_{2m+1}(t')\xi(t') + \sum_{m,n=0}^{\infty}\frac{i^{2n+1}}{(2n+1)!}\frac{(-i)^{2m}}{2m!}Z_{2n+1}(t)Z_{2m}(t')\xi(t')\Big)\Big\rangle\Big\langle R^{\xi}[t';\xi(\tau)]\Big\rangle.\end{aligned} \tag{A24}$$

From Eq. (A22) it follows,

$$\frac{d}{dt'}\Phi[t-t';\nu(\tau)] = \Big\langle\Big(\sum_{n=0}^{\infty}\frac{i^n}{n!}Z_n(t)\Big)\Big(\frac{d}{dt'}\sum_{m=0}^{\infty}\frac{(-i)^m}{m!}Z_m(t')\Big)\Big\rangle. \tag{A25}$$

A short computation yields,

$$\frac{d}{dt'}\Phi[t-t';\nu(\tau)] = -i\nu(t')\Big\langle\Big(\sum_{n=0}^{\infty}\frac{i^n}{n!}Z_n(t)\Big)\Big(\sum_{m=0}^{\infty}\frac{(-i)^m}{m!}Z_m(t')\xi(t')\Big)\Big\rangle$$
$$= -i\nu(t')\Big(\Big\langle\sum_{m,n=0}^{\infty}\frac{i^{2n}}{2n!}\frac{(-i)^{2m+1}}{(2m+1)!}Z_{2n}(t)Z_{2m+1}(t')\xi(t') + \sum_{m,n=0}^{\infty}\frac{i^{2n+1}}{(2n+1)!}\frac{(-i)^{2m}}{2m!}Z_{2n+1}(t)Z_{2m}(t')\xi(t')\Big)\Big\rangle. \quad \text{(A26)}$$

Using this result in Eqs. (A21) and (A22), we obtain

$$\Big\langle \exp\Big\{i\int_0^{t-t'} d\tau\xi(\tau)\nu(\tau)\Big\}R[t';\xi(\tau)]\Big\rangle = \Phi[t-t';\nu(\tau)]\big\langle R[t';\xi(\tau)]\big\rangle - \frac{1}{i\nu(t)\chi(0)}\frac{d}{dt'}\Phi[t-t';\nu(\tau)]\big\langle R^{\xi}[t';\xi(\tau)]\big\rangle, \quad t\geq t'. \quad \text{(A27)}$$

□

Corollary 1 *Let $\varphi(t) = D\int_0^t \xi(\tau)d\tau$. Then, for an arbitrary functional, $R[t;\xi(\tau)]$, the following correlation splitting holds:*

$$\big\langle e^{i\varphi(t)}e^{-i\varphi(t')}R[t;\xi(\tau)]\big\rangle = \Phi(t-t')\big\langle R[t;\xi(\tau)]\big\rangle - \frac{1}{iD\sigma^2}\frac{d}{dt'}\Phi(t-t')\big\langle R^{\xi}[t;\xi(\tau)]\big\rangle, \quad \text{(A28)}$$

where $\Phi(t-t') = \langle e^{i\varphi(t)}e^{-i\varphi(t')}\rangle$ and $\sigma^2 = \chi(0)$.

Appendix B: Population distribution inside acceptor band

The population distribution, $\langle\rho_{EE}(t)\rangle$, inside acceptor band can be found as solution of the following differential equation, obtained from Eqs. (30) - (32) in the same approximation as Eqs. (48) - (49). The population density inside of the band is given by $\rho(E) = \langle\rho_{EE}\rangle\varrho(E)$. For $\rho(E)$ we obtain the following differential equation:

$$\frac{d}{dt}\rho(E,t) = \gamma_1(t)\big\langle\rho_{11}(t)\big\rangle - \gamma_2(t)\langle\bar\rho_{22}(t)\rangle, \quad \text{(B1)}$$

where

$$\gamma_1(t) = v^2 e^{-\alpha(E-E_0)^2}\int_0^t dt' e^{-\bar p^2(t-t')^2/4}\Big(e^{i(E_1-E)(t-t')} + e^{-i(E_1-E)(t-t')}\Big). \quad \text{(B2)}$$
$$\gamma_2(t) = v^2 e^{-\alpha(E-E_0)^2}\int_0^t dt' e^{-\bar p^2(t-t')^2/4}e^{-t'^2/4\alpha}\Big(e^{i\varepsilon(t-t')}e^{i(E_0-E)t} + e^{-i\varepsilon(t-t')}e^{-i(E_0-E)t}\Big), \quad \text{(B3)}$$

and we set $\bar p = \sqrt{2}D\sigma$. Performing the integration, we obtain

$$\gamma_1(t) = \frac{\sqrt{\pi}v^2}{\bar p}\exp\Big(-\alpha\Delta_0^2 - \frac{\Delta_1^2}{\bar p^2}\Big)\Big(\mathrm{erf}\Big(\frac{\bar p t}{2} + i\frac{\Delta_1}{\bar p}\Big) - \mathrm{erf}\Big(i\frac{\Delta_1}{\bar p}\Big) + \text{c.c.}\Big),$$
$$\tilde\gamma_2(t) = \frac{\sqrt{\pi}v^2}{p}\exp\Big(-\alpha\Delta_0^2 - i\Delta_0 t - \frac{t^2}{4\alpha}\Big)\Big(\Big(\exp\frac{(t/2\alpha - i\varepsilon)^2}{p^2}\Big)\mathrm{erf}\Big(\frac{pt}{2} - \frac{t/2\alpha - i\varepsilon}{p}\Big) - \exp\Big(-\frac{\varepsilon^2}{p^2}\Big)\mathrm{erf}\Big(\frac{i\varepsilon}{p}\Big)\Big) + \text{c.c.}, \quad \text{(B4)}$$

where $\Delta_0 = E_0 - E$ and $\Delta_1 = E_1 - E$.

$$\tilde\gamma_2(t) = \frac{A\sqrt{\pi}}{p}\exp\Big(-i\Delta_0 t - \frac{t^2}{4\alpha}\Big)\Big(\Big(\exp\frac{(t/2\alpha - i\varepsilon)^2}{p^2}\Big)\mathrm{erf}\Big(\frac{pt}{2} - \frac{t/2\alpha - i\varepsilon}{p}\Big) - \exp\Big(-\frac{\varepsilon^2}{p^2}\Big)\mathrm{erf}\Big(\frac{i\varepsilon}{p}\Big)\Big), \quad \text{(B5)}$$

The solution of Eq. (B1) can be written as,

$$\langle\bar\rho_{EE}(t)\rangle = \int_0^t(\gamma_1(\tau) + \gamma_2(\tau))\langle\rho_{11}(\tau)\rangle d\tau - \int_0^t \gamma_2(\tau)d\tau. \quad \text{(B6)}$$

Using the asymptotic formula, $\langle\rho_{11}(t)\rangle \sim e^{-\Gamma_1 t}$, where

$$\Gamma_1 = \frac{2\sqrt{\pi}\, v^2}{p} e^{-\varepsilon^2/p^2}, \tag{B7}$$

and expanding the limits of integration in the last integral as, $\tau \to \infty$, we obtain

$$\rho(E) = \rho(E,t)\big|_{t\to\infty} = \int_0^\infty (\gamma_1(t) + \gamma_2(t)) e^{-\Gamma_1 t} dt - \int_0^\infty \gamma_2(t) dt. \tag{B8}$$

The computation yields

$$\rho(E) = \frac{\sqrt{\pi/2}v^2}{D\sigma}\Big(\frac{\delta}{2\sqrt{\pi}\Gamma_1}\Psi_1(E) + \Psi_2(E) - \Psi_0(E)\Big) e^{-4\pi(E_0-E)^2/\delta^2}, \tag{B9}$$

where

$$\Psi_0(E) = 2\mathrm{Re}\Big(w\Big(\frac{E - E_0 - \varepsilon}{\sqrt{2}D\sigma}\Big) w\Big(\frac{2\sqrt{\pi}(E - E_0)}{\delta}\Big)\Big), \tag{B10}$$

$$\Psi_1(E) = 2\mathrm{Re}\Big(w\Big(\frac{E - E_0 - \varepsilon + i\Gamma_1}{\sqrt{2}D\sigma}\Big)\Big), \tag{B11}$$

$$\Psi_2(E) = 2\mathrm{Re}\Big(w\Big(\frac{E - E_0 - \varepsilon + i\Gamma_1}{\sqrt{2}D\sigma}\Big) w\Big(\frac{2\sqrt{\pi}(E - E_0 + i\Gamma_1)}{\delta}\Big)\Big). \tag{B12}$$

Here, $w(z) = e^{-z^2}\mathrm{erfc}(-iz)$ is a complex complementary error function [5].

To proceed further, we use the approximated formula [5]:

$$w(z) \approx \begin{cases} 1, & |z| \ll 1 \\ \dfrac{i}{\sqrt{\pi}z}, & |z| \gg 1 \end{cases} \tag{B13}$$

In the limit, $\bar{p} \ll \Gamma_1$ (weak noise) and $\delta \ll \varepsilon$ (narrow zone), we obtain the leading term as,

$$\langle\bar{\rho}_{EE}\rangle \sim \frac{2v^2}{\Gamma_1^2 + (E - E_1)^2}. \tag{B14}$$

For $\sqrt{\Gamma_1^2 + (\varepsilon + E_0 - E)^2} \ll \bar{p}$ (strong noise), we find

$$\langle\bar{\rho}_{EE}\rangle \sim \frac{2v^2}{\bar{p}\Gamma_1}\Big(1 + \frac{2{\Gamma_1}^2}{\Gamma_1^2 + (E - E_0)^2}\Big). \tag{B15}$$

Details of computation

Here we present the details of calculations for $\langle\rho_{EE}\rangle$. We start with some auxiliary integrals [6, 7]

$$\int e^{-(ax^2+bx+c)} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}} e^{(b^2-4ac)/(4a)} \mathrm{erf}(\sqrt{a}\,x + \frac{b}{2\sqrt{a}}), \tag{B16}$$

$$\int_0^\infty e^{-px}\mathrm{erf}(cx + b) dx = \frac{1}{p}\mathrm{erf}(b) + \frac{1}{p}\exp\Big(\frac{p^2 + 4pbc}{4c^2}\Big)\mathrm{erfc}\Big(b + \frac{p}{2c}\Big), \tag{B17}$$

$$\int e^{-px}\mathrm{erf}(cx + b) dx = -\frac{1}{p} e^{-px}\mathrm{erf}(cx + b) + \frac{1}{p}\exp\Big(\frac{p^2 + 4pbc}{4c^2}\Big)\mathrm{erf}\Big(cx + b + \frac{p}{2c}\Big). \tag{B18}$$

Performing integration in (B2) with help of (B16), we obtain

$$\begin{aligned}\gamma_1(t) =& |V|^2 \varrho_0 \int_0^t e^{-\bar{p}^2\tau^2/4}\Big(e^{i(E_1-E)\tau} + e^{-i(E_1-E)\tau}\Big) d\tau \\ =& A\exp\Big(-\frac{\Delta_1^2}{\bar{p}^2}\Big)\Big(\mathrm{erf}\Big(\frac{\bar{p}t}{2} + i\frac{\Delta_1}{\bar{p}}\Big) - \mathrm{erf}\Big(i\frac{\Delta_1}{\bar{p}}\Big) + \mathrm{c.c.}\Big),\end{aligned} \tag{B19}$$

where $\Delta_1 = E_1 - E$ and $A = 2v^2/(\bar{p}\delta)$.

Next step is to find $I_1(t) = \int_0^t \gamma_1(\tau) e^{-\Gamma_1 \tau d\tau}$. Using (B18), we obtain

$$I_1(t) = -\frac{A}{\Gamma_1} e^{-\Gamma_1 t}\Big(\mathrm{erfc}\Big(\frac{\bar{p}t}{2} + \frac{i\Delta_1}{\bar{p}}\Big) - \mathrm{erfc}\Big(\frac{i\Delta_1}{\bar{p}}\Big)\Big) + \frac{A}{\Gamma_1}\Big(\exp\Big(\frac{(\Gamma_1 + i\Delta_1)^2}{\bar{p}^2}\Big)\Big(\mathrm{erfc}\Big(\frac{\bar{p}t}{2} + \frac{\Gamma_1 + i\Delta_1}{\bar{p}}\Big) - \mathrm{erfc}\Big(\frac{\Gamma_1 + i\Delta_1}{\bar{p}}\Big)\Big) + \mathrm{c.c.}\Big). \tag{B20}$$

To calculate $\gamma_2(t)$, it is convenient to define a new function,

$$\tilde{\gamma}_2(t) = \frac{A}{\varrho_0}\sqrt{\frac{\alpha}{\pi}} \int_{-\infty}^{\infty} dE' \varrho(E') \int_0^t dt' e^{-\bar{p}^2(t-t')^2/4} e^{-i(E_1 - E)t} e^{i(E_1 - E')t'}, \tag{B21}$$

so that $\gamma_2 = \tilde{\gamma}_2 + \tilde{\gamma}_2^*$. Performing integration over band, we obtain

$$\begin{aligned}\tilde{\gamma}_2(t) =& A \int_0^t dt' e^{-t'^2/4\alpha} e^{-\bar{p}^2(t-t')^2/4} e^{i(E-E_1)t} e^{i\varepsilon t'} \\ =& A e^{-t^2/4\alpha - i\Delta_0 t} \int_0^t dt' e^{-p^2\tau^2/4} e^{(t/2\alpha - i\varepsilon)\tau} \\ =& \frac{A\sqrt{\pi}}{p} \exp\Big(-i\Delta_0 t - \frac{t^2}{4\alpha}\Big)\Big(\Big(\exp\frac{(t/2\alpha - i\varepsilon)^2}{p^2}\Big)\mathrm{erf}\Big(\frac{pt}{2} - \frac{t/2\alpha - i\varepsilon}{p}\Big) - \exp\Big(-\frac{\varepsilon^2}{p^2}\Big)\mathrm{erf}\Big(\frac{i\varepsilon}{p}\Big)\Big),\end{aligned} \tag{B22}$$

where $\Delta_0 = E_0 - E$.

Using (B16), we obtain,

$$\int_0^t \tilde{\gamma}_2(\tau) d\tau = \frac{A\sqrt{\pi}}{p} \int_0^t d\tau \exp\Big(-i\Delta_0\tau - \frac{\tau^2}{4\alpha}\Big)\Big(\exp\frac{(\tau/2\alpha - i\varepsilon)^2}{p^2}\Big)\mathrm{erf}\Big(\frac{p\tau}{2} - \frac{\tau/2\alpha - i\varepsilon}{p}\Big) + \frac{A\sqrt{\pi}}{p}\exp\Big(-\frac{\varepsilon^2}{p^2}\Big)\mathrm{erf}\Big(\frac{i\varepsilon}{p}\Big)\Big(\mathrm{erf}\Big(\frac{t}{2\alpha} + i\sqrt{\alpha}\Delta_0\Big) - \mathrm{erf}(i\sqrt{\alpha}\Delta_0)\Big). \tag{B23}$$

Introducing a new variable $\tau = t - t'$, we can rewrite (B21) as,

$$\tilde{\gamma}_2(t) = \frac{2v^2}{\sqrt{\pi}\bar{p}\varrho_0\delta} \int_{-\infty}^{\infty} dE' \varrho(E') e^{-i(E'-E)t} \int_0^t e^{-\bar{p}^2\tau^2/4} e^{-i(E_1 - E')\tau} d\tau, \tag{B24}$$

Then, integrating over τ, we obtain

$$\tilde{\gamma}_2(E,t) = \frac{2v^2}{\sqrt{\pi}\bar{p}\varrho_0\delta} \int_{-\infty}^{\infty} dE' \varrho(E') e^{i\Delta' t} \exp\Big(-\frac{\Delta'^2_1}{\bar{p}^2}\Big)\Big(\mathrm{erf}\Big(\frac{\bar{p}t}{2} + i\frac{\Delta'_1}{\bar{p}}\Big) - \mathrm{erf}\Big(i\frac{\Delta'_1}{\bar{p}}\Big)\Big), \tag{B25}$$

where $\Delta' = E - E'$ and $\Delta'_1 = E_1 - E'$.

Next step is to calculate the contributions: $I_{1,2}(\Gamma_1) := \int_0^\infty \gamma_{1,2}(E,t) e^{-\Gamma_1 t} dt$ and $I_0 := \int_0^\infty \gamma_2(E,t) dt$. As can be seen, $I_0 = I_2(0)$. We start with $I_1(\Gamma_1)$. Using (B18), we obtain

$$I_1(\Gamma_1) = \frac{2v^2}{\bar{p}\Gamma_1\delta} \exp\Big(\frac{(\Gamma_1 + i\Delta_1)^2}{\bar{p}^2}\Big)\Big(\mathrm{erfc}\Big(\frac{\Gamma_1 + i\Delta_1}{\bar{p}}\Big) + \mathrm{c.c.}\Big). \tag{B26}$$

This can be recast as follows: $I_1(\Gamma_1) = \tilde{I}_1(\Gamma_1) + \tilde{I}_1^*(\Gamma_1)$,

$$\tilde{I}_1(\Gamma_1) = \frac{2v^2}{\bar{p}\Gamma_1\delta} w\Big(\frac{\Delta_1 + i\Gamma_1}{\bar{p}}\Big), \tag{B27}$$

where $w(z) = e^{-z^2}\mathrm{erfc}(-iz)$ [5].

To calculate $I_2(\Gamma_1)$, we rewrite it as, $I_2(\Gamma_1) = \tilde{I}_2(\Gamma_1) + \tilde{I}_2^*(\Gamma_1)$,

$$\tilde{I}_2(\Gamma_1) = \int_0^\infty \tilde{\gamma}_2(E,t) e^{-\Gamma_1 t} dt = \int_{-\infty}^{\infty} dE' \varrho(E') f(\Gamma_1), \tag{B28}$$

where

$$f(\Gamma_1) = \frac{2v^2}{\sqrt{\pi}\bar{p}\varrho_0\delta}\, e^{-\Delta'^2_1/\bar{p}^2} \int_0^\infty dt\, e^{-(\Gamma_1 - i\Delta')t}\left(\mathrm{erf}\Big(\frac{\bar{p}t}{2} + i\frac{\Delta'_1}{\bar{p}}\Big) - \mathrm{erf}\Big(i\frac{\Delta'_1}{\bar{p}}\Big)\right). \tag{B29}$$

Performing the integration, we obtain

$$f(\Gamma_1) = \frac{2v^2}{\sqrt{\pi}\bar{p}\varrho_0\delta}\frac{1}{\Gamma_1 - i\Delta'}\exp\left(\frac{(\Gamma_1 + i\Delta_1)^2}{\bar{p}^2}\right)\mathrm{erfc}\Big(\frac{\Gamma_1 + i\Delta_1}{\bar{p}}\Big) = \frac{\Gamma_1 \tilde{I}_1(\Gamma_1)}{\sqrt{\pi}\varrho_0(\Gamma_1 - i\Delta')}. \tag{B30}$$

Inserting $f(\Gamma_1)$ into Eq. (B28), we obtain

$$\tilde{I}_2(\Gamma_1) = \tilde{I}_1(\Gamma_1)\frac{\Gamma_1}{\sqrt{\pi}\varrho_0}\int_{-\infty}^{\infty}\frac{\varrho(E')dE'}{\Gamma_1 - i\Delta'}. \tag{B31}$$

For the Gaussian density of electron states in the acceptor band, $\varrho(E) = \varrho_0 e^{-\alpha(E-E_0)^2}$, we have

$$\int_{-\infty}^{\infty}\frac{\varrho(E')dE'}{\Gamma_1 - i\Delta'} = \int_{-\infty}^{\infty}\frac{\varrho_0 e^{-\alpha(E'-E_0)^2}dE'}{\Gamma_1 - i(E - E')}. \tag{B32}$$

This integral can be calculated using the following relation [5]:

$$w(z) = \frac{i}{\pi}\int_{-\infty}^{\infty}\frac{e^{-t^2}dt}{z - t} \quad (\Im z > 0). \tag{B33}$$

A computation yields

$$\int_{-\infty}^{\infty}\frac{\varrho_0 e^{-\alpha(E'-E_0)^2}dE'}{\Gamma_1 - i(E - E')} = \pi\varrho_0 w\Big(\frac{2\sqrt{\pi}(i\Gamma_1 - \Delta_0)}{\delta}\Big). \tag{B34}$$

Using this result, we obtain

$$\tilde{I}_2(\Gamma_1) = \sqrt{\pi}\Gamma_1\tilde{I}_1(\Gamma_1)w\Big(\frac{2\sqrt{\pi}(i\Gamma_1 - \Delta_0)}{\delta}\Big) = \frac{2\sqrt{\pi}v^2}{\bar{p}\delta}w\Big(\frac{\Delta_1 + i\Gamma_1}{\bar{p}}\Big)w\Big(\frac{2\sqrt{\pi}(i\Gamma_1 - \Delta_0)}{\delta}\Big). \tag{B35}$$

Since $I_0 = I_2(0)$, we obtain

$$\tilde{I}_0 = \frac{2\sqrt{\pi}v^2}{\bar{p}\delta}w\Big(\frac{\Delta_1 + i\Gamma_1}{\bar{p}}\Big)w\Big(\frac{2\sqrt{\pi}(i\Gamma_1 - \Delta_0)}{\delta}\Big). \tag{B36}$$

Performing the integration with $\Phi(t - t')$ taken from Eq. (57), after some calculations, we obtain

$$\langle\rho_{EE}\rangle \sim b\Big(\frac{\delta}{2\sqrt{\pi}\Gamma_1}\Psi_1(E) + \Psi_2(E) - \Psi_0(E)\Big), \tag{B37}$$

where $b = \sqrt{\pi}v^2\varrho_0/\bar{p}$, $\bar{p} = \sqrt{2}D\sigma$ and

$$\Psi_0(E) = 2\mathrm{Re}\left(w\Big(\frac{E - E_1}{\bar{p}}\Big)w\Big(\frac{2\sqrt{\pi}(E - E_0)}{\delta}\Big)\right), \tag{B38}$$

$$\Psi_1(E) = 2\mathrm{Re}\left(w\Big(\frac{E - E_1 + i\Gamma_1}{\bar{p}}\Big)\right), \tag{B39}$$

$$\Psi_2(E) = 2\mathrm{Re}\left(w\Big(\frac{E - E_1 + i\Gamma_1}{\bar{p}}\Big)w\Big(\frac{2\sqrt{\pi}(E - E_0 + i\Gamma_1)}{\delta}\Big)\right). \tag{B40}$$

Appendix C: Estimation of the approximations in the main text

While the function, $W(\tau)$, is defined explicitly in terms of the special functions, the analytical expression for $Z(\tau)$ is unknown. Thus, we restrict ourselves to numerical simulations in order to estimate the validity of the approximation for the results obtained in the main text of the paper. In Figs. 6 and 5, we present the plots of the function, $Z(\tau)$, for the parameters used in Figs. 9 – 16, 19 and 21 – 26 in the paper. In the title to each figure below, the number of the corresponding figure in the main text is shown. Only main parameters for identification of the corresponding figures in the main text are presented below in figure captions for functions, $Z(\tau)$. These results allow us to obtain estimates for the accuracy of our approximation.

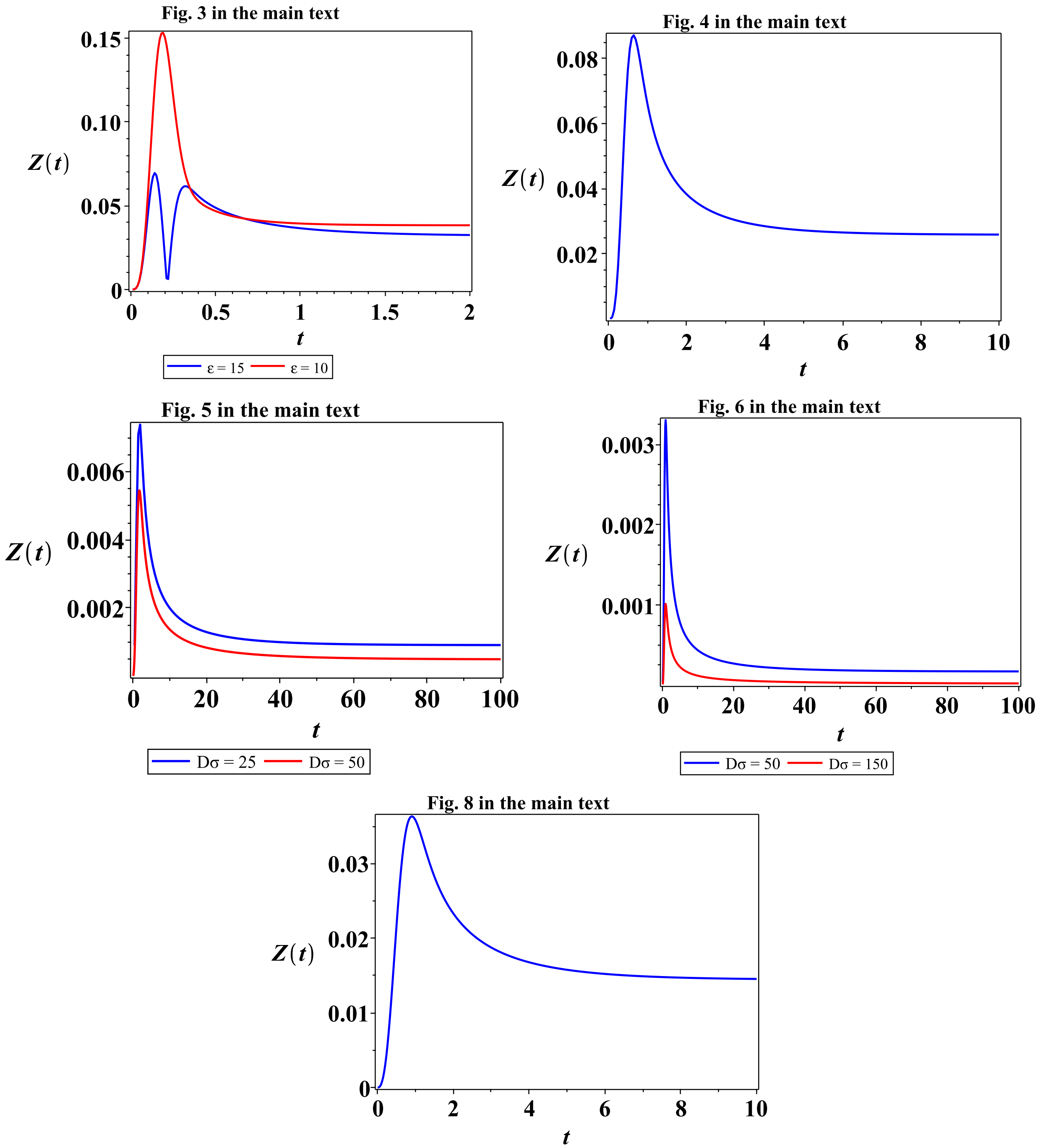

FIG. 4: (Color online) Dependence of Z on time t. Estimates are made for the results presented in Figs. 3 – 6 and 8 in the main text.

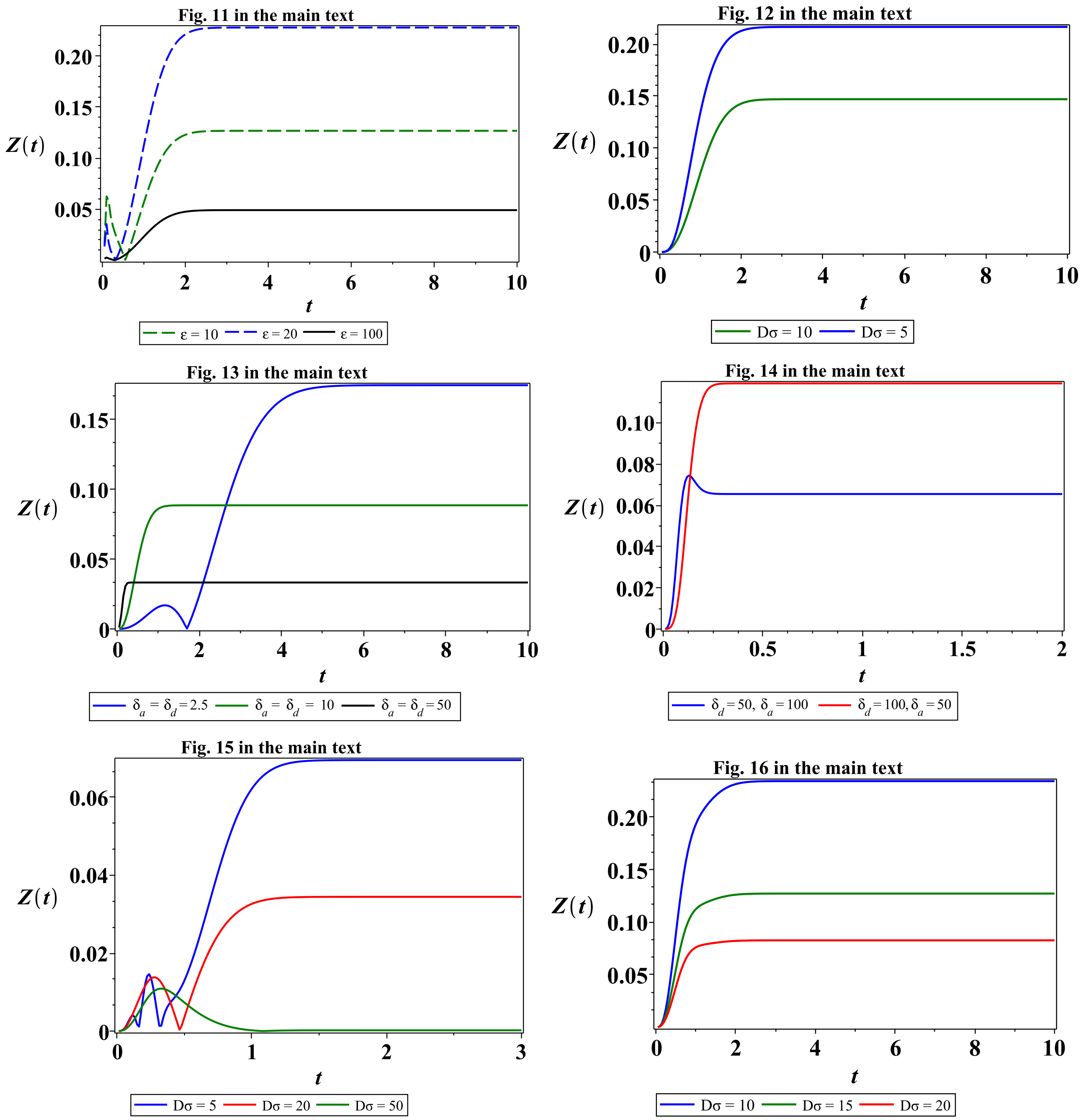

FIG. 5: (Color online) Dependence of Z on time t. Estimates are made for the results presented in Figs. 11 – 16 in the main text.

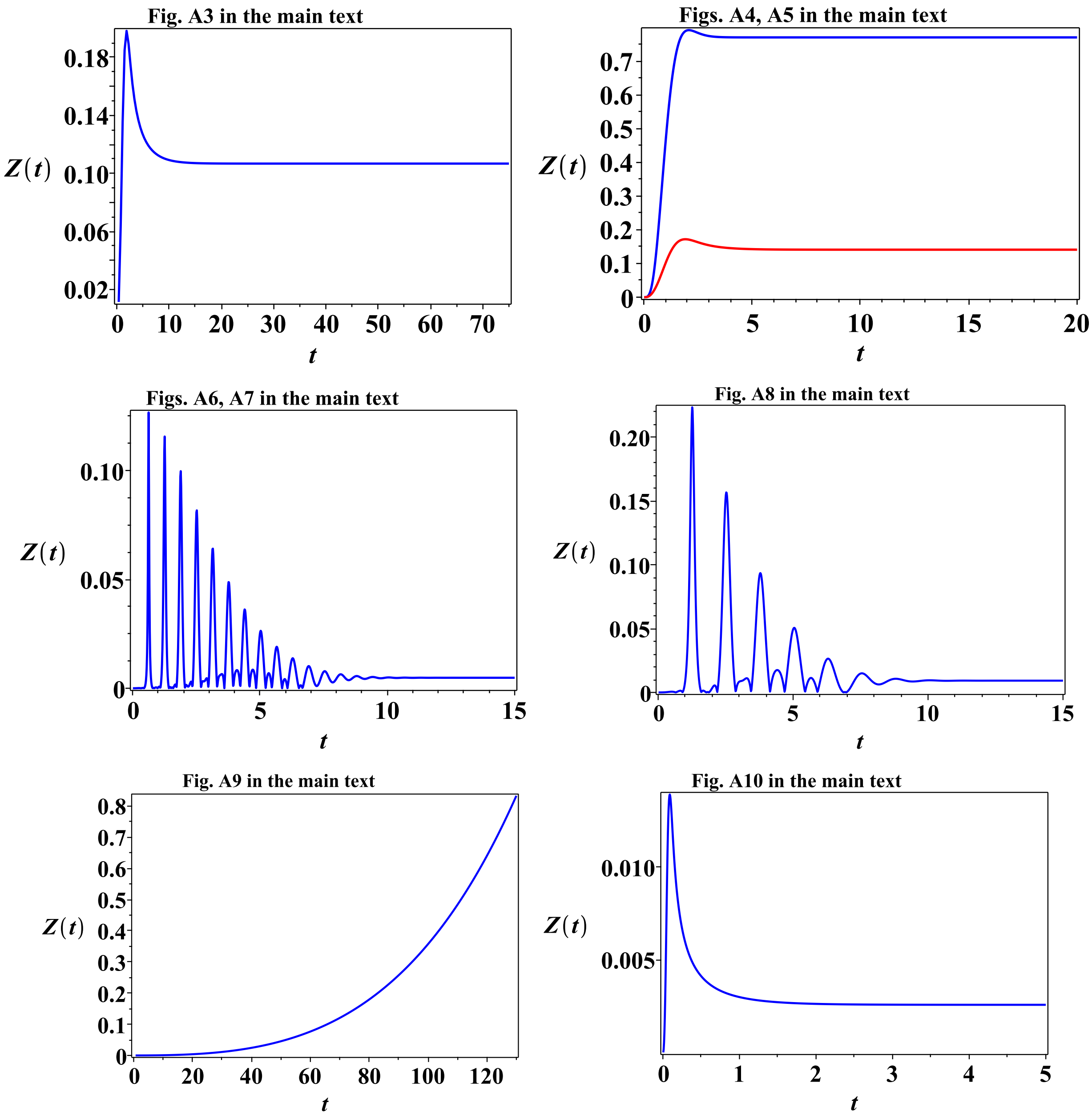

FIG. 6: (Color online) Dependence of Z on time t. Estimates are made for the results presented in Figs. A6 – A10 in the main text (Appendix A).