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CORRECTION

Correction to: Completely positive dynamical semigroups and quantum resonance theory

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Correction. The bound (2.23) in Theorem 2.1 has to be replaced by

$$\left|R_{\lambda,t}(X)\right| \leq C\left(|\lambda| + \lambda^2 t\right) \mathrm{e}^{-\lambda^2(1+O(\lambda))\gamma t}\, \|X\|. \tag{0.1}$$

Implication. The difference is that in reality we can only show $\lambda^2 t$ on the right side, instead of the $|\lambda|^3 t$ as announced in the published paper. The remainder (0.1) is still asymptotically exact (vanishing as $t \to \infty$), and our result still proves that the dynamics is approximated, asymptotically exactly, by a CPT semigroup. But for times $t \approx 1/\lambda^2$, the approximation is not guaranteed to be small.

Nevertheless, as will be discussed elsewhere in more detail, we can obtain a proof of Theorem 2.1, exactly as stated in the published paper, if instead of allowing all observables $X \in \mathcal{B}(\mathcal{H}_{\mathrm{S}})$, we restrict to X *which commute with* H_{S}. Such observables determine the dynamics of the system populations (diagonal elements of the reduced density matrix). This means that the true population dynamics is approximated by a CPT semigroup dynamics, uniformly in time to accuracy $O(\lambda)$, and the approximating Markovian dynamics is also asymptotically exact.

The original article can be found online at https://doi.org/10.1007/s11005-017-0937-z.

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Proof of the correction (0.1). An estimate for the remainder term in the proof of Theorem 2.1, as given in the paper, contains a mistake. Namely, in footnote 5 [after Eq. (3.44)], the real phase of the exponential was omitted. The correct estimate is

$$\left|\mathrm{e}^{\mathrm{i}t(e+\lambda^2 a_{e,j})} - \mathrm{e}^{\mathrm{i}t(\widetilde{e}+\lambda^2\widetilde{\lambda}_{\widetilde{e},j})}\right| \leq C\lambda^2 t\,\mathrm{e}^{-\lambda^2(1+O(\lambda))\gamma t}. \tag{0.2}$$

The corrected bound (0.2) yields (0.1) by the argument in the paper. We get (0.2) as follows,

$$\begin{aligned}\left|\mathrm{e}^{\mathrm{i}t(e+\lambda^2 a_{e,j})} - \mathrm{e}^{\mathrm{i}t(\widetilde{e}+\lambda^2\widetilde{\lambda}_{\widetilde{e},j})}\right| &= \mathrm{e}^{-t\lambda^2 \mathrm{Im} a_{e,j}}\left|1-\mathrm{e}^{\mathrm{i}t(\widetilde{e}-e+\lambda^2(\widetilde{\lambda}_{\widetilde{e},j}-a_{e,j}))}\right| \\ &= \mathrm{e}^{-t\lambda^2 \mathrm{Im} a_{e,j}}\left|\int_0^{t(\widetilde{e}-e+\lambda^2(\widetilde{\lambda}_{\widetilde{e},j}-a_{e,j}))}\mathrm{e}^{\mathrm{i}z}\mathrm{d}z\right| \\ &\leq \mathrm{e}^{-t\lambda^2 \mathrm{Im} a_{e,j}}\; t\left|\widetilde{e}-e+\lambda^2(\widetilde{\lambda}_{\widetilde{e},j}-a_{e,j})\right|\mathrm{e}^{t\lambda^2|\mathrm{Im}(\widetilde{\lambda}_{\widetilde{e},j}-a_{e,j})|}.\end{aligned}$$

Using in the last inequality that $\mathrm{Im} a_{e,j} = (1+O(\lambda))\gamma$, that $\widetilde{\lambda}_{\widetilde{e},j} - a_{e,j} = O(\lambda)$ and that

$$\widetilde{e} - e = O(\lambda^2) \tag{0.3}$$

gives (0.2). We note that by Lemma 3.1 in the published paper, the bound (0.3) above is seemingly only $O(\lambda)$. However, since $\rho_{\mathrm{S},\beta,\lambda} - \rho_{\mathrm{S},\beta,0} = O(\lambda^2)$ (the linear term vanishes since the interaction has vanishing average in the reservoir vacuum state), we have $\widetilde{H}_{\mathrm{S}} - H_{\mathrm{S}} = O(\lambda^2)$. The $O(\lambda)$ bounds in Lemma 3.1 are thus actually $O(\lambda^2)$ bounds and (0.3) holds.