well as the resonance energies $\varepsilon_e^{(s)}$ to lowest order in the interaction, see Theorem 2.2. Those quantities are the key ingredients entering the dynamics which we describe in Theorem 2.4.

Let $\underline{\sigma}, \underline{\tau}$ be spin configurations of the form (1.8). Then

$$\varphi_{\underline{\sigma},\underline{\tau}} = \varphi_{\sigma_1\tau_1} \otimes \cdots \otimes \varphi_{\sigma_N\tau_N} \quad \text{with} \quad \varphi_{\sigma\tau} = \varphi_\sigma \otimes \varphi_\tau \in \mathbb{C}^2 \otimes \mathbb{C}^2 \tag{2.7}$$

is an eigenvector of L_S with eigenvalue $e(\underline{\sigma},\underline{\tau}) = \sum_j B_j(\sigma_j - \tau_j)$. The genericness condition (1.21) implies that if $\varphi_{\underline{\sigma},\underline{\tau}}$ and $\varphi_{\underline{\sigma}',\underline{\tau}'}$ are eigenvectors associated to the same eigenvalue, then $\sigma_j - \tau_j = \sigma_j' - \tau_j'$ for all j. If $\sigma_j - \tau_j = \pm 2$ then $\sigma_j = \pm 1$ and $\tau_j = \mp 1$ are determined uniquely, while if $\sigma_j - \tau_j = 0$, then there are two choices, $\sigma_j = \tau_j = \pm 1$. Consequently, an orthonormal basis of eigenvectors of L_S associated to a given eigenvalue e can be constructed as follows. Take any *one* eigenvector $\varphi_{\underline{\sigma},\underline{\tau}}$ associated to e and adjoin all linearly independent vectors $\varphi_{\underline{\sigma}',\underline{\tau}'}$ with the property $\{\sigma_j - \tau_j = 0\} \Longleftrightarrow \{\sigma_j' - \tau_j' = 0\}$. Thus, with each eigenvalue e we associate the number

$$N_0(e) = \{\text{number of indices } j \text{ s.t. } \sigma_j = \tau_j \text{ in any } (\underline{\sigma},\underline{\tau}) \text{ with } e(\underline{\sigma},\underline{\tau}) = e\}, \tag{2.8}$$

and the degeneracy of the eigenvalue e of L_S is $d(e) = 2^{N_0(e)}$. To each eigenvalue e of L_S there corresponds a unique sequence of $N_0(e)$ indices indicating the locations j at which $\sigma_j = \tau_j$ for all $\underline{\sigma}, \underline{\tau}$ associated with e. In other words, given e there is a unique sequence $\{\mu_k\}_{k=1}^{N_0(e)}$

$$1 \leqslant \mu_1 < \mu_2 < \cdots < \mu_{N_0(e)} \leqslant N, \tag{2.9}$$

having the property that any eigenvector $\varphi_{\underline{\sigma},\underline{\tau}}$ associated to e satisfies

$$\sigma_j = \tau_j \Longleftrightarrow j \in \{\mu_k : k = 1, \dots, N_0(e)\}. \tag{2.10}$$

Given an energy difference e (1.13), and a sequence $\underline{\varrho} = (\varrho_j)_{j=1}^{N_0(e)}$, $\varrho_j \in \{+1, -1\}$, we set

$$\delta_e^{(\underline{\varrho})} = \lambda_1^2 \cancel{[x_1}(e) + \mathrm{i}y_1(e)\cancel{]} + \lambda_2^2[x_2(e) + \mathrm{i}y_2(e)] + \sum_{j=1}^{N_0(e)} z_j^{\varrho_j}, \tag{2.11}$$

μ_j

where

$$\cancel{x_1(e) = -e_0 \, \mathrm{P.V.}\langle g_1, \omega^{-1} g_1\rangle \sum_{\{j:\sigma_j=\tau_j\}} \sigma_j,} \tag{2.12}$$

$$y_1(e) = \frac{\pi e_0^2}{2\beta}\gamma_+, \tag{2.13}$$

$$x_2(e) = -2\sum_{\{j:\sigma_j\neq\tau_j\}} \sigma_j \; \mathrm{P.V.} \int_{\mathbb{R}} u^2 \mathcal{G}_2(2u) \coth(\beta|u|) \frac{1}{u - B_j} \, \mathrm{d}u, \tag{2.14}$$

$$y_2(e) = 2\pi \sum_{\{j:\sigma_j\neq\tau_j\}} B_j^2 \mathcal{G}_2(2B_j) \coth(\beta B_j), \tag{2.15}$$

$$z_j^{\pm} = \frac{1}{2}\left[\mathrm{i}b_j(c_j + 1) \pm \sqrt{-b_j^2(c_j+1)^2 + 4a[a - \mathrm{i}b_j(c_j - 1)]}\right], \tag{2.16}$$

$+$

with

$$a = -\lambda_1^2 e_0 \, \mathrm{P.V.}\langle g_1, \omega^{-1} g_1\rangle, \quad b_j = 4\pi\lambda_2^2 \frac{B_j^2 \mathcal{G}_2(2B_j)}{\mathrm{e}^{2\beta B_j} - 1}, \quad c_j = \mathrm{e}^{2\beta B_j}, \tag{2.17}$$

and

$$e_0 = e_0(e) = \sum_{j=1}^{N} (\sigma_j - \tau_j), \quad \mathcal{G}_k(u) = \int_{S^2} g_k(|u|, \Sigma)|^2 \, \mathrm{d}\Sigma, \quad \gamma_+ = \lim_{u\to 0_+} u\mathcal{G}_1(u). \tag{2.18}$$

The form factors g_1, g_2 (see (1.4)) are represented in spherical coordinates in (2.18) and P.V. stands for principal value. Note that e_0 is the same for all spin configurations $\underline{\sigma}, \underline{\tau}$ associated to the same energy $e = e(\underline{\sigma}, \underline{\tau})$. This follows from the genericness of the magnetic field, (1.21), see paragraph after (2.7). We show in Theorem 3.5 that $\mathrm{Im} z_j^{\pm} \geqslant 0$. Let us define the vectors