MEMORIAL UNIVERSITY OF NEWFOUNDLAND
Department of Mathematics and Statistics

AMAT-2120 $\hspace{5cm}$ Fall 2004

Assignment 3. Due at 10:00am Friday October 29

Project "Means".

The arithmetic mean of two given real numbers x and y is

$$A(x, y) = \frac{x + y}{2}.$$

Their geometric mean is (assuming $x, y > 0$)

$$G(x, y) = \sqrt{xy},$$

and their harmonic mean is the number $H(x, y)$ such that

$$\frac{1}{H(x, y)} = \frac{1}{2} \left(\frac{1}{x} + \frac{1}{y} \right).$$

Finally, there is the so called Arithmetic-Geometric Mean (AGM) with a rather tricky definition. Let

$$x_0 = x, \qquad y_0 = y.$$

Then, for each $i = 1, 2, \ldots$ let

$$x_i = A(x_{i-1}, y_{i-1}), \qquad y_i = G(x_{i-1}, y_{i-1}).$$

The common limit (its existence is a theorem)

$$AG(x, y) = \lim_{i \to \infty} x_i = \lim_{i \to \infty} y_i$$

is the AGM of x and y.

Write a program to compute these various means. Parameters provided by user will be: the values of x, y; what kind of mean is to be computed. In the case of AGM your program may take an optional argument — the number of iterations to approximate the limit.

Write a design specification for your program, specifying which arguments are to be typed in the command line and which are to be requested interactively.

Use modular design for your program. The `main` function should contain no more that 10 lines of code (except for comments).

Test your program. In verifying the AGM calculations, you can use Maple's function `GaussAGM(..., ...)`.