COURSE DESCRIPTION

Introduction to geometric group theory.

1 Rationale

This is an introductory course to geometric group theory, a relatively new field with roots in combinatorial group theory and low dimensional topology. The philosophy of the area is to understand the relation between algebraic properties of discrete groups and geometric properties of spaces on which the groups act. Geometric group theory has grown into a major field contributing to the solution of long standing open problems in different areas of mathematics, and it is currently a very active area of research.

This course will introduce basic terminology of geometric group theory, some standard proof techniques, as well as a general overview of types of questions and problems that researchers in this field engage into. The course is designed for first year graduate students, and it only assumes familiarity with group theory, metric spaces and point set topology at the undergraduate level.

Brief overview of the course: The basic idea is to consider finitely generated groups and view them as geometric objects, usually through geometric actions on metric spaces, or on their Cayley graphs. In this framework, certain geometric and combinatorial properties of these spaces and/or graphs can be interpreted as algebraic properties of the group, and viceversa; such properties are called quasi-isometric invariants. We will look at some examples of these type invariants, for instance, we will discuss why being nilpotent, abelian or finite are geometric notions. Then we will focus on a particular class of groups known as Gromov hyperbolic groups which can be introduced in geometric terms, and which have been intensively studied in the last thirty years. A standard class of examples of hyperbolic groups is known as the (high genus) surface groups which we will study in some detail. We will discuss some of the properties of hyperbolic groups as, for example, the solvability of the word problem via Dehn's algorithm. The course will conclude with a brief overview of some invariants as boundaries at infinity and ends of groups.

2 Textbook for the course

> Löh, Clara. *Geometric Group Theory: An Introduction.* Springer International Publishing, 2017.

The textbook is available online at Memorial University library website.

3 Temptatively Course outline

1. Part I. Groups (Weeks 1 and 2)
   (a) Groups via generators and relators.

   (b) Cayley graphs of groups.
   (c) Products, extensions, free products and amalgamated products.

2. Part II. Group actions on metric spaces, and Quasi-isometry. (Weeks 2 and 3)
   (a) Group actions on metric spaces.
   (b) Groups as geometric objects, Cayley graphs.
   (c) Free groups and actions on trees.
   (d) Ping pong lemma, and free subgroups of matrix groups.
   (e) The integral special linear groups $SL(n, \mathbb{Z})$.
   (f) Quasi-isometry and the Svarkc-Milnor Lemma.
   (g) Examples of quasi-isometry invariants.

3. Part III. Nilpotent groups and polynomial growth. (Weeks 4 and 5)
   (a) Nilpotent groups.
   (b) Growth of types and quasi-isometry.
   (c) The Heisenberg group.
   (d) Polynomial growth implies virtual nilpotence.
   (e) Gromov's polynomial growth theorem, overview.
   (f) Quasi-isometric rigidity of free abelian groups.

4. Part IV. Hyperbolic Groups. (Weeks 6, 7 and 8)
   (a) Hyperbolic spaces.
   (b) Stability of quasi-geodesics, and quasi-isometry invariance of hyperbolicity.
   (c) Hyperbolic groups and examples.
   (d) The word problem on hyperbolic groups (Dehn's algorithm).
   (e) Surface groups and small cancellation groups
   (f) Brief overview of properties of hyperbolic grops (finite order elements, centralizers, etc)
   (g) Quasiconvex subgroups.

5. Part V. Geometry at infinity. (Weeks 9 and 10)
   (a) Ends of groups and Stallings theorem.
   (b) The Gromov boundary of a hyperbolic group.
   (c) Free groups in hyperbolic groups via Ping-pong.

6. Part VI. Student presentations and final remarks. (Weeks 11 and 12)

4 Prerequisites

There are no prerequisites.

However, the course assumes familiarity with abstract algebra, and point set topology at the undergraduate level.