MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

MIDTERM TEST | **Pure Mathematics 3370** | OCTOBER 26, 1998

Marks

[4] 1. Let $\{F_n\}_{n=1}^{\infty}$ be the Fibonacci sequence. Prove that $F_n = \dfrac{\alpha^n - \beta^n}{\sqrt{5}}, n \geq 1$, where α and β are the positive and negative roots, respectively, of the equation $x^2 - x - 1 = 0$.

[7] 2. (a) Define a Mersenne number. What is the first Mersenne number that is not prime?

(b) Prove that there are arbitrarily large gaps between successive prime numbers.

(c) State clearly the Prime Number Theorem.

[4] 3. (a) Solve the Diophantine equation $189x + 1220y = 12,131$.

(b) Find the positive solutions, if any.

[2] 4. Find the last two digits of the number 367^{4764}. (You should use the fact that $\phi(100) = 40$.)

[8] 5. Prove **TWO** of the following:

(a) Prove Fermat's (Little) Theorem, that is, prove $a^{p-1} \equiv 1 \pmod{p}$, if p is a prime, and $p \nmid a$. If you use Euler's Theorem, prove it!

(b) If p is a prime and $p \equiv 1 \pmod{4}$, prove that the congruence $x^2 \equiv -1 \pmod{p}$ has a solution. (You may use Wilson's Theorem without proof.)

(c) If p is prime prove that $(p-1)! \equiv -1 \pmod{p}$.

(d) Prove that any prime congruent to 1 modulo 4 can be written as the sum of two squares. (Hint: Wilson's Theorem is needed. Any other results used should be proved.)

---

[25]