

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

MIDTERM EXAM

Pure Mathematics 3370

OCTOBER 25, 2004

Marks

- [7] 1. (a) Define the gcd of two integers a and b .
(b) Solve the Diophantine equation

$$461x + 142y = 4789.$$

- (c) Find the positive solution pairs, if any.

- [7] 2. (a) List 50 consecutive composite integers.
(b) State the Prime Number Theorem.
(c) Find the remainder when 319^{566} is divided by 23.

- [3] 3. If $\{f_n\}_{n=1}^{\infty}$ is the Fibonacci sequence, prove that $f_n < \alpha^n$ for all $n \geq 1$, where $\alpha = \frac{1+\sqrt{5}}{2}$.

- [3] 4. Prove **ONE** of the following:

- (a) If $2^n - 1$ is prime, prove that $N = 2^{n-1}(2^n - 1)$ is even and perfect.
(b) If p is prime and $x^2 \equiv -1 \pmod{p}$ has a solution, prove that $p = 2$ or $p \equiv 1 \pmod{4}$.
(c) If $(a, b) = 1$, prove that all the solutions of $ax + by = n$ are given by $x = x_0 + bt$ and $y = y_0 - at$ where (x_0, y_0) is a particular solution of $ax + by = n$.