MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

FINAL EXAM | **Pure Mathematics 3370** | FALL 1999

Marks

[3] 1. (a) If $a \mid bc$ and $(a, b) = 1$, prove that $a \mid c$.

[3] (b) Solve the Diophantine equation $25x + 11y = 557$.

[2] (c) Find the positive solutions, if any.

[3] 2. (a) Prove that any composite integer n has a prime factor $\leq \sqrt{n}$.

[2] (b) List 50 consecutive composite numbers.

[3] (c) Give a formula to generate all the primitive Pythagorean triples and list 6 such triples.

[3] 3. (a) Find the last two digits of 9^{99999}.

[3] (b) Find the common solution of the congruences $x \equiv 16 \pmod{41}$, $x \equiv 2 \pmod{7}$, and $x \equiv 2 \pmod{15}$.

[2] 4. (a) Define a *primitive root* modulo a positive integer m.

[2] (b) How many primitive roots are there modulo $m = 125$?

[3] (c) If a has order h modulo m, prove that $h \mid \phi(m)$.

[3] 5. (a) **Either:** Prove that a rational prime $p \equiv 1 \pmod{4}$ is not a Gaussian prime.
OR: Prove, using the Either part, that such a prime can be written as the sum of two squares of rational integers.

[3] (b) Factor the Gaussian integer $14(23 - 15i)$.

[5] 6. Prove **ONE** of the following theorems:

(a) If $(a, m) = 1$ and $m \geq 1$, prove that $a^{\phi(m)} \equiv 1 \pmod{m}$.

(b) If p is a prime then $(p - 1)! \equiv -1 \pmod{p}$.

(c) Every even perfect number is of the form $N = 2^{n-1}(2^n - 1)$ with $2^n - 1$ a prime.

[40]