MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Fall 2005	**Pure Mathematics 3370** **Assignment 7**	Due: Friday November 4, 2005

---

1. Compute $\phi(10^5)$, $\phi(256)$, $\phi(9901)$, and $\phi(462^3)$.

2. If precisely the primes 3, and 7 divide n prove that $\phi(n) = \dfrac{4n}{7}$.

3. If $a \mid b$, prove that $\phi(a) \mid \phi(b)$. Is the converse true?

4. Find the smallest primitive root g modulo 54. Write the numbers 41 and 23 as powers of g, and compute 41×23 modulo 54 using these powers.

5. If a has order h modulo m, and $ab \equiv 1 \pmod{m}$, prove that b has order h.

6. (a) Which m have primitive roots for $m = 77,\ 2012,\ 1622$, and $498002 = 2(499)^2$?

   (b) How many primitive roots modulo m are there for those m above which have primitive roots?

7. (a) Prove $\displaystyle\prod_{\substack{p|m \\ or \\ p|n}}(1 - \frac{1}{p}) \prod_{\substack{p|m \\ and \\ p|n}}(1 - \frac{1}{p}) = \prod_{p|m}(1 - \frac{1}{p}) \prod_{p|n}(1 - \frac{1}{p})$.

   (b) Let $(m, n) = g$. Prove that $\phi(mn)\phi(g) = g\phi(m)\phi(n)$. (Hint: Use the formula $\phi(u) = u \displaystyle\prod_{p|u}(1 - \frac{1}{p})$.)

   (c) Hence, prove $\phi(mn) = \phi(m)\phi(n)$ if and only if $(m, n) = 1$.

The Final Exam in PM 3370 is Wednesday, December 14, 2004 at 9am in HH-3017.