MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

FALL 2005

Pure Mathematics 3370
Assignment 5

DUE: WEDNESDAY OCTOBER 19, 2005

Marks

1. Being Number Theory students, you should be able to give clear arguments for the following quick ways to check divisibility by 3, 9 and 7. Use the notation $N = (a_n a_{n-1} ... a_2 a_1 a_0)_{10}$ with $0 < a_n \leq 9$, $0 \leq a_i \leq 9$, for $0 \leq i < n$ to mean

$$N = 10^n a_n + 10^{n-1} a_{n-1} + \cdots 10^2 a_2 + 10 a_1 + a_0.$$

[3] (a) Prove that a number is divisible by 9 if and only if the sum of its digits is divisible by 9. (Hence conclude the result for 9 replaced by 3.)

[3] (b) Given the number 2492, double the units digit and subtract it from the number formed by the other digits. We get $249 - 2 \times 2 = 245$. Repeating this algorithm we get $24 - 2 \times 5 = 14$. Since 14 is clearly divisible by 7, the original number 2492 must be divisible by 7. Prove this rule for checking divisibility by 7.

[3] 2. Show that $2^{50} + 3^{50}$ is divisible by 13. (Use Fermat's (Little) Theorem.)

[3] 3. Determine the remainder when 6^{3725} is divided by 41.

[3] 4. Find the inverse of 200 modulo 4001.

[3] 5. If x and y are odd, prove that $x^2 + y^2$ cannot be a perfect square.

[3] 6. Prove that $a^2 - 29b^2 = 14$ has no integer solutions. (Hint: If there is a solution, then it is also a solution to a congruence for some (small) modulus m.)

[4] 7. For m and n any two integers, prove that $mn(m^4 - n^4)$ is always divisible by 30. (Hint: Use Fermat's Little Theorem. Consider each prime factor of 30.)

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[25]

Don't forget the Mid-Term Test, Monday, October 24, 2005