MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

FALL 2005 | **Pure Mathematics 3370** **Assignment 2** | DUE: FRIDAY SEPTEMBER 23, 2005

Marks

1. Let $F_n = 2^{2^n} + 1$.

[3] (a) Prove that $\Pi_{0 \leq k < n} F_k = F_n - 2$ for $n \geq 1$.

[4] (b) Hence prove that $(F_m, F_n) = 1$ for $m \neq n$.

[3] (c) Hence prove that there are infinitely many primes.

2. In your solution set we proved that $\alpha^n = f_{n-1} + \alpha f_n$ where α is *any* root of $x^2 - x - 1 = 0$. (You should study this proof!) Use this result to

[3] (a) derive the *Binet* formula

$$f_n = \frac{\alpha^n - \beta^n}{\sqrt{5}},$$

where α, β are the roots of $x^2 - x - 1 = 0$, α being the larger root; (In class we proved the Binet formula by induction.)

[3] (b) evaluate α^{15}.

[6] 3. For each of the pair of numbers (a, b), find the gcd and find x and y such that $ax + by = \text{gcd}$: (a) (61358,2090) (b) (24168,6555).

[3] 4. If $a \mid b$ and $b \mid a$, prove that $a = \pm b$.

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[25]