MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Fall 2005	**Pure Mathematics 3370** **Assignment 1**	Due: Friday September 16, 2005

Marks

[3] 1. Recall that the *binomial coefficient* $\binom{m}{k}$ is defined to be $\dfrac{m!}{k!(m-k)!}$, where $m \geq k \geq 0$, $k! = k(k-1)\cdots 3\cdot 2\cdot 1$ and $0! = 1$. Prove, by induction, that $\binom{2n}{n} \geq \dfrac{4^n}{n+1}$ if $n \geq 1$.

[6] 2. Let $\{f_n\}_{n=1}^{\infty}$ be the Fibonacci sequence.

(a) Prove that $f_n^2 - f_{n-1}f_{n+1} = (-1)^{n-1}$ for $n \geq 2$.

(b) Prove that $\alpha^{n-2} \leq f_n \leq \alpha^{n-1}$ for all $n \geq 1$, where $\alpha = \dfrac{1+\sqrt{5}}{2}$.

[3] 3. (a) Prove by mathematical induction that for n odd, $x^{n-1} - x^{n-2} + \ldots + x^2 - x + 1 = \dfrac{x^n+1}{x+1}$.

[3] (b) If $2^a + 1$ is prime, prove that $a = 2^m$ for some $m \in N$. (Hint: Use part (a).)

[4] 4. Use the $\log_{10}$ function on your calculator to find the number of digits in the largest known prime (the largest one, not the one in your Course Notes). Justify your answer.

[6] 5. In a letter to Euler in 1742, Goldbach stated that Statement A: *Every integer greater than 5 is the sum of three primes.* Euler replied that this was equivalent to Statement B: *Every even integer greater that or equal to 4 is the sum of two primes.* Show that the two statements are equivalent. (Hint: To prove A implies B, consider $n+2$, and to prove B implies A, consider n even and the n odd.)

The assignments are on my home page: **http://www.math.mun.ca/∼drideout**.

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