Some Useful Information for Lab 4A

The hardest part of Lab 4A (Conics and Light) is iterating the trajectory of a point (x_0, y_0) inside an ellipse in the direction $(\cos t_1, \sin t_1)$. For that problem, it seems best to write the equations of lines in parametric form. The parametric form of the ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$$x = a\cos t, \quad y = b\sin t, \quad 0 \le t \le 2\pi.$$

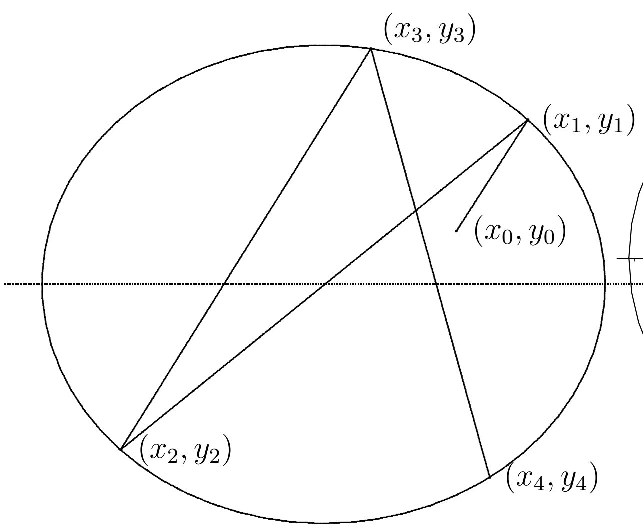
The initial trajectory (line) has a parametric equation of the form:

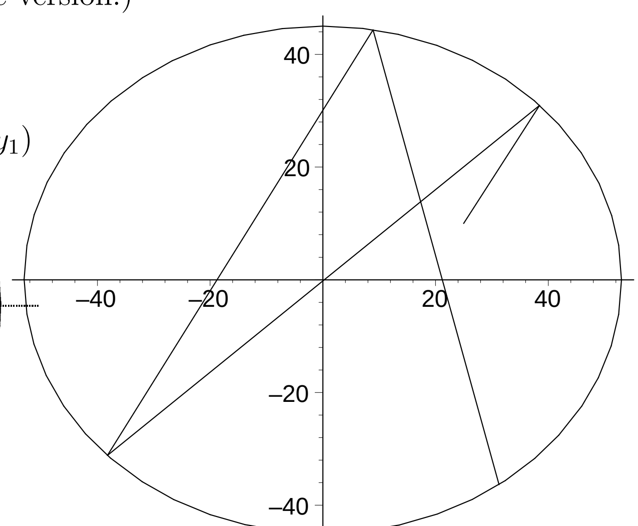
$$x = x_0 + u\cos t_1, \quad y = y_0 + u\sin t_1.$$

To compute (x_1, y_1) you need to substitute x and y in the equation of the ellipse, and solve the quadratic equation for u. Pick the solution with the positive square root, say. The next challenge is to get the coordinates of the next point (x_2, y_2). The line from (x_1, y_1) to (x_2, y_2) has the parametric form

$$x = x_1 + u\cos t_2, \quad y = y_1 + u\sin t_2. \tag{1}$$

The angle t_2 is a simple function of α_1 and t_1, where $\tan\alpha_1$ is the slope of the tangent line at (x_1, y_1). (Be careful with vertical tangent lines.) After substituting (1) in the equation of the ellipse, the resulting equation simplifies to a linear equation in u. (Why is this?) The graph of four iterations for the ellipse with $a = 53, b = 45, (x_0, y_0) = (25, 10)$ and $t_1 = 1.0$ is given below. (The second graph is the Maple version.)





A larger version that you can use to determine the angles is on the second page: