Applied Mathematics 2130

Lab 2006W-4A
Conics and Light

The very same old Archimedes, whose π algorithm some of you studied in Lab 2, once set a Roman fleet on fire by concentrating the sun's rays on the ships through the use of a great concave mirror. At least that's what a popular historical anecdote says.

In this lab we ask you to study how a concave mirror works. For simplicity, let us assume that the action takes place in Flatland — that is, in a two-dimensional world, and that the mirror is a conic section: an ellipse, a parabola, or a branch of a hyperbola.

The task is to investigate the trajectory of a light beam emitted from a source inside or on the surface of the mirror. The initial direction must be given. Assume that the beam is infinitely thin, so that it propagates along a straight line in free space and obeys the law of reflection when it hits the mirror.

A computer program that you will write must calculate consecutive reflection points (as many as specified by the user) and directions of propagation between adjacent points. Then you will plot the mirror and the trajectory of light.

We suggest that you concentrate primarily on the case of an elliptic mirror. Before writing a program it is necessary, of course, to prepare a mathematical background: how you describe an ellipse; what exactly the law of reflection says; how to find tangent and normal lines to the ellipse at a given point; how to launch a ray in a given direction.

If your program is to deal with parabolic and hyperbolic mirrors as well, make sure it takes into account a possibility of a ray going free to infinity. Therefore the actual number of reflections can be less than that specified by the user.

We expect you to make some experimental observations and, if possible, generalize them and support them by mathematical proofs. With regard to where and what to look at, here are some hints:

1. Take the source at a focus of an ellipse and observe the trajectory.
2. For a general source, iterate the trajectory for a fairly long time. What is the "envelope" of the multiply reflected rays? (The envelope is also called a *caustic*; it is the locus of hottest spots, where the rays "accumulate".)
3. With a parabolic mirror, consider a ray coming from infinity parallel to the parabola's axis. Find the trajectory. Graph two, three, ...such trajectories on the same figure.

By the end of this work, you will probably figure out Archimedes' idea (if only it was his dream) about burning an enemy's fleet. What type of mirror would he need for that? And, by the way, what's the shape of your satellite antenna dish?

Following the *Suggested Report Format*, prepare a report that addresses the issues mentioned above. Include suitable illustrative graphics supporting your conclusions. As always, supporting mathematical reasoning is essential.