Applied Mathematics 2130

Lab 2006W–2A [1]

Bob is a little league hockey coach. There is a big tournament coming up and he figures his young team has a good chance of making it to the semi-finals. This alone will be a great boost to his team's morale.

On Bob's team is a young player named Tina, a.k.a. "*slapshot*" Tina. She earned her nickname by being recognized as the kid with the fastest slapshot this league has ever seen. Tina however has two little problems: her accuracy with the puck is not too good, and she can't skate too well yet. As a coach, Bob wants to capitalize on Tina's skills as best as he can. For this tournament, his advice to Tina is the following:

> Whenever she gets the puck, she is to skate up the ice in a straight line (parallel to the sides) from wherever she is and shoot at the point where the angle on the net is largest.

This way, her problem with inaccuracy is minimized and the skating is as simple as it can be. The problem is Bob can't figure out how far up the ice Tina should go to in order to have the largest angle possible on the net. This is where you come in. Your task is to solve this problem for Tina (and Bob).

Let's call the solution set, that is, the locus of points where Tina should skate to, the "*slapshot curve*". You should discover everything you can about this curve. The problem can be treated by purely mathematical means; it is in fact an optimization problem of a kind you should be familiar with from Calculus. On the other hand, you may find it easier to explore various positions on Tina's trajectory and numerically find the one that maximizes the angle subtended by the net. Whatever way you choose to determine the optimal slapshot positions, we ask you to write a computer program which, given any position Tina might be in on the ice, will identify where she should skate to and launch her best slapshot ever.

There are a few things to keep in mind. First, there is a goal crease, so if Tina is headed straight at the goal, she cannot go all the way up to the goal line. As it turns out, Bob also coaches soccer and it would seem that whatever works for ice hockey ought to be adaptable for soccer too. Thus your solution should not be restricted to the strict geometry of North American ice hockey.

When your analysis and computer programs are completed, prepare a report (following the *Suggested Report Format*) providing a full description of your solution. All key terms should be clearly defined. Supporting graphics illustrating the geometric quantities of interest should be included as well. If you choose to use Maple for this lab, prepare an accompanying worksheet that will convincingly convey your findings to its reader. Supporting mathematical reasoning is essential. This, combined with a few truly informative illustrations is all that is required.

[1] The original idea is from Grant Woods.