Applied Mathematics 2130
Lab 2006W–1

The purpose of this laboratory is to explore the effects that various algebraic manipulations induce on the graph of a function. By definition, the graph of a function f is the set in $\mathbb{R}^2$ given by

$$\{(x, y) \;:\; y = f(x), \quad x \in D \subseteq \mathbb{R}\}.$$

There are numerous ways, from simple to most peculiar, in which we can create a new graph from that of a given function f. The most basic approach is to pick real numbers a, b, c, and d and consider the new functions:

$$(1)\quad f(ax), \qquad (2)\quad bf(x), \qquad (3)\quad f(x-c), \qquad (4)\quad f(x)+d.$$

In this assignment we want you to investigate the effects of each of the above manipulations on the original graph. You should use a variety of values for a, b, c, and d in order to fully illustrate these effects. Also we ask you to describe mathematically what happens in general to the set of ordered pairs $\{(x, y)\}$ that constitute the original graph.

Methodology

As a first step, you will write a program which will produce a file consisting of pairs of numbers, that is, the points (x, y) that constitute the graph of $y = f(x)$. This file will be passed to a graphics generation program to generate your graph (consult the course manual). Your program should accept as data the endpoints of the interval of interest and the number of points to be plotted. The program should be sufficiently simple so that it can be be changed easily as you consider different functions. Since you will be graphing some functions with vertical asymptotes, your program should be able to find these asymptotes and be able to stay within a suitably chosen bound for the y–values.

Once your data generation program is written, check it by plotting two functions whose graphs are completely familiar to you.

Now that your program is operational, we want you to practice. Select and graph the following functions. Choose suitable domains to illustrate all notable features of the graphs.

$$y = \tfrac{1}{6}x^3 - \tfrac{1}{2}x^2 - \tfrac{3}{2}x + 3 \qquad\qquad y = \tfrac{1}{5}x^5 - 2x$$

$$y = \tfrac{1}{4}x^4 + \tfrac{1}{2}x^3 - \tfrac{3}{4}x^2 - x + 1 \qquad\qquad y = \tfrac{1}{5}x^{2/3}(10 - x)$$

$$y = \frac{x}{x^2+2} \qquad\qquad y = \frac{1}{x^2-4}$$

$$y = 2 + x^{2/3} \qquad\qquad y = \cos x$$

$$y = |x-1| + |x+1| - \sqrt{|x^2-1|} \qquad\qquad y = \cot x$$

$$y = x^2(1 - \ln x) \qquad\qquad y = 4x^2 e^x$$

Once you have been successful in plotting each of the functions on the indicated interval, perform a series of experiments on some of the functions to determine the effect that each of the four processes has on the graph of the original function. (Note that this will be easy to do if your program accepts $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{d}$ as input.)

The report

Select four functions from the above list, including at least one discontinuous function, and a fifth function (one not listed here already) whose geometric properties you consider to be **enlightening** in the present context.

Manipulate the graphs of these functions and illustrate the effects of operations (1)–(4) on the graphs. Experiment as much as you like, but one to two illustrations per manipulation is plenty for your report. Are there clearly defined relationships between the original $\boldsymbol{f(x)}$ and those to which you have applied operations (1)–(4)? In some or all of the chosen functions you should draw the graph of a primary function and the altered function on the same set of axes. Explain clearly how a general point $\boldsymbol{(x, y)}$ gets altered by each of the above manipulations on the original graph. Illustrate this by drawing an arrow from a suitably chosen point(s) of the original graph to the alterated graph.

Prepare a report following the *Suggested Report Format* supporting your conclusions regarding the effect of operations (1)–(4) on the graphs. Supporting mathematical reasoning is essential. This, combined with a **few** truly informative illustrations is all that is required.

A closing remark. Remember that good mathematics requires precise definitions. An important idea here is the *shape of a graph*. Unfortunately, this is not a well-defined mathematical concept. As you carry out the suggested experiments and as you begin writing your report, you might wish to do some investigation into the meaning of the *shape of a graph*. You might also want to establish a procedure one might use to assess whether two graphs have the same shape. (Your calculus graphing techniques can be considered here.)