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Einstein's 'spooky action' common in large quantum systems

Mathematicians identify entanglement threshold, adding parameters to quantum computing efforts

Entanglement is a property in quantum mechanics that seemed so unbelievable and so lacking in detail that, 66 years ago this spring, Einstein called it "spooky action at a distance."

But a mathematician at Case Western Reserve University and two of his recent PhD graduates show entanglement is actually prevalent in large quantum systems and have identified the threshold at which it occurs.

The finding holds promise for the ongoing push to understand and take advantage of the property. If harnessed, entanglement could yield super high-speed communications, hack-proof encryptions and quantum computers so fast and powerful they would make today's supercomputers look like adding machines in comparison.

The mathematicians don't tell us how entanglement works, but were able to put parameters on the property by combining math concepts developed for a number of different applications during the last five decades. In a nutshell, the researchers connected the math to properties of quantum mechanics—the otherworldly rules that best apply to atomic and subatomic particles—to describe physical reality.

"There have been indications that large subgroups within quantum systems are entangled," said Stanislaw Szarek, mathematics professor at Case Western Reserve and an author of the study. "Our contribution is to find out exactly when entanglement becomes ubiquitous."

Szarek worked with Guillaume Aubrun, assistant professor of mathematics at Université Claude Bernard Lyon 1, France, and Deping Ye, assistant professor of mathematics and statistics at Memorial University of Newfoundland, Canada. Their work is published online in the Early View section of *Communications on Pure and Applied Mathematics*.

The behaviors of materials down at the level of atoms are often strange, but entanglement borders on our concepts of sorcery. For example, if two electrons spinning in opposite directions are entangled, when one changes direction, the other immediately changes, whether the electrons are side by side, across the room or at opposite ends of the universe.

Other particles, such as photons, atoms and molecules, can also become entangled, but taking advantage of the property requires more than a pair or handful.

Szarek, Aubrun and Ye focused on large quantum systems—large groups of particles that have the potential for use in our world.

They found that, in systems in a random state, two subsystems that are each less than one-fifth of the whole are generally not entangled. Two subsystems that are each greater than one-fifth of the whole typically are entangled. In other words, in a system of 1,000 particles, two groups that are smaller than 200 each typically won't be entangled. Two groups larger than 200 each typically will.

Further, the research shows, "the change is abrupt when you reach the threshold of about 200," Szarek said.

The team also calculated the threshold for positive partial transpose, or PPT, a property related to entanglement. If the property is violated, entanglement is present.

"From these two perspectives, the calculations are very precise." Szarek said.

Harsh Mathur, a physics professor at Case Western Reserve whom Szarek consulted to better understand the science, said, "Their point is entanglement is hard to create from a small system, but much easier in a large system."

"And the thing that Einstein thought was so weird is the rule rather than the exception," Mathur added.

The researchers used mathematics where analysis, algebra and geometry meet, Szarek said. The math applies to hundreds, thousands or millions of dimensions.

"We put together several things from different parts of mathematics, like a puzzle, and adapted them," he said. "These are mathematical tools developed largely for aesthetical reasons, like music."

The ideas—concepts developed in the 1970s and 1980s and more recently— turned out to be relevant to the emerging quantum information science.

"We have found there is a way of computing and quantifying the concept of quantum physics and related it to some calculable mathematical quantities," Szarek continued. "We were able to identify features and further refine the description, which reduces the questions about the system to calculable and familiar looking mathematical quantities."

So, if entanglement is more common in large quantum systems, why aren't they being used already?

"In the every day world, it's hard to access or create large quantum mechanical systems to do meaningful quantum computations or for communications or other uses," Mathur said. "You have to keep them isolated or they decohere and behave in a classical manner. But this study gives some parameters to build on."

Szarek will continue to investigate mathematics and quantum information theory while attending the Isaac Newton Institute for Mathematical Sciences in Cambridge, England in the fall. He will work with computer scientists and quantum physicists during a semester-long program called Mathematical Challenges in Quantum Information. He received a $101,000 National Science Foundation grant to participate.

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Quantum entanglement isn't only spooky, you can't avoid it

By Brian Dodson

June 10, 2013

Quantum entanglement is the key to quantum computing, cryptography, and numerous other real-world applications of quantum mechanics. It is also one of the strangest phenomena in the Universe, overcoming barriers of space and time and knitting the entire cosmos into an integrated whole. Scientists have long thought that entanglement between two particles was a rare and fleeting phenomenon, so delicate that exposure of the particles to their surroundings would quickly destroy this linkage. Now mathematicians at Case Western University have shown that entanglement between parts of large systems is the norm, rather than being a rare and short-lived relationship.

Entanglement is one of the strangest predictions of quantum mechanics. Two objects are entangled if their physical properties are undefined but correlated, even when the two objects are separated by a large distance. No mechanism for entanglement is known, but so far experiments universally show that nonlocal entanglement is real. When two entangled particles are subjected to the influence of a surrounding environment, their interactions with the surroundings cause the entanglement to "leak out" into the surroundings, so it is more difficult to detect and use, but it does not disappear.

Entanglement is clearly subtle, but how common is it in the real world of macroscopic objects? A new research paper from Professor Stanislaw Szarek's mathematics group at Case Western

Reserve University addresses this question, and finds that entanglement is ubiquitous in large objects.

Their analysis is essentially statistical, where the quantum probabilities are studied using the tools of geometric functional analysis, a field of mathematics well suited for addressing problems associated with very large numbers of dimensions.

Systems of a few particles will tend to lie close to a pure state, a state in which none of the internal particles are entangled with each other. The particles of such a system will show essentially no sign of being entangled. You can create a state of a few particles in which the particles are entangled, but these states are quite unusual.

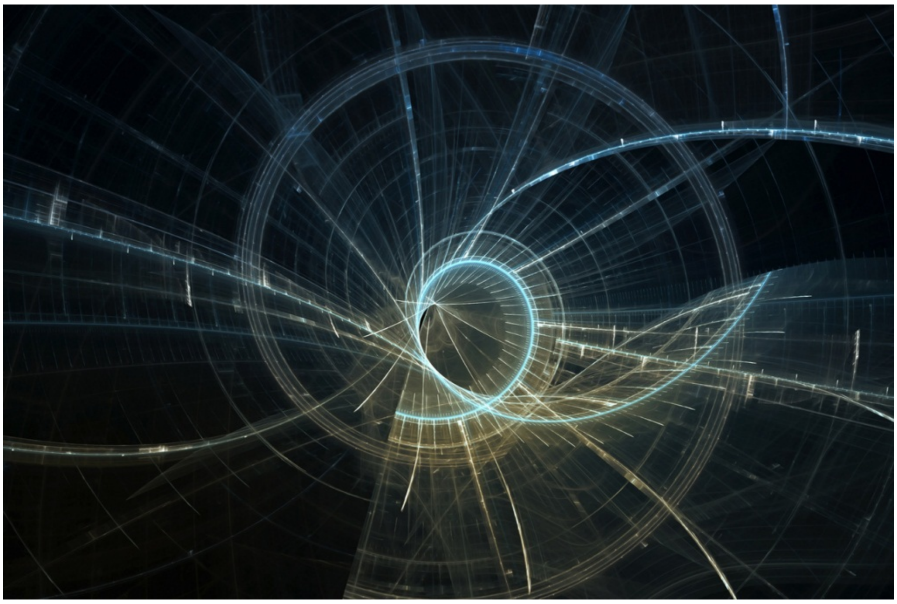
When you consider larger systems, perhaps having thousands (or trillions) of particles, the quantum description is essentially the same, but the way the quantum attributes of the system scale with size changes the probabilities considerably. Now the pure states form only a very small portion of the possible quantum states, and as a result, the more probable behavior is that parts of the system are entangled with each other.

Szarek's team also considered the entanglement of subsystems of an entangled system. If you choose two particles from a system, the chance that they are entangled is very small; in fact, vanishingly small in the limit of very large systems. On the other hand, if you split the system in two, these halves are almost certain to be entangled with each other.

In the end, their analysis shows that in systems having large numbers of particles, a pair of tiny subsystems tend not to be entangled with each other, but a pair of large subsystems tend to be entangled. If you consider two subsystems each having fewer than about one-fifth of the total number of particles in the overall system, the subsystems are almost certainly not entangled with each other. If the two subsystems are larger than one-fifth of the original system, they are almost certainly entangled. The abrupt change in entanglement behavior is characteristic of the geometry of high-dimensional spaces.

The result shows that everyday objects are so constructed that their parts are entangled with each other, and are also entangled with most everything with which they have previously interacted. This is an interesting result, particularly for those who think of the Universe in holistic terms, but does this holism have any observable consequences? This is a very difficult question, to which we don't yet have a practical answer.

Large-scale entanglement guides how our world evolves, often in crucial ways. However, predicting how a specific action might change that evolution appears impossible, at least in any practical sense. Such prediction simply requires too much knowledge about the microscopic state of the world. One might say, facetiously, that magic works, but usually has no real and/or predictable effect. At least, within quantum mechanics.



Quantum Entanglement Common In Large Systems?

A property of the universe so strange that our man himself, Einstein, called it "…spooky action at a distance…". In an entangled system, two particles e.g. photos will interact and then be separated with exactly the same quantum state; that is to say, same momentum, spin etc. and then if one particle is affected, say its spin reversed, the same will be observed for the other particle. This in itself is strange enough, but it gets weirder. Even when separated by a distance, be it 1 km or 1 trillion, the two particles will change at exactly the same time; the transfer of state is instantaneous. Yet, get this; new research conducted by three researchers at Case Western Reserve University have shown that this property of entangled matter is common in larger systems.

The mathematicians didn't set out to explain how quantum entanglement works, but rather to find the threshold at which it becomes a common property. By connecting quantum mechanics and some very high level maths developed in the last five decades they were able to show that in a system of a random state, if we were to separate it into more than five subsystems, you would not observe two entangled states; however if you took the system and split it into five or less states, you would likely find two subsystems which were entangled. For example in a system of 1000 particles, two subsystems of less than 200 particles would not likely be entangled, but two subsystems larger than 200 typically will. The change around the threshold of 200 is substantial. The calculations they conducted were very precise and drew on of areas of mathematics which had previously only been developed for aesthetical reasons, but have now found a use in the real world.

The lead mathematician, Stanislaw Szarek, will be attending a semester long program at Cambridge in order to continue this investigation into this strange development.

This fantastically strange find has shown the world that there is promise in the new area of study, quantum information science and that one day we may be able to use this science to create hack-proof encryptions and computers so fast they make our best supercomputers look like adding machines from the 1800s.