## PMAT 3331 – Projective Geometry Winter 2003

## Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 3:00 pm on Friday March 28, 2003.
- Please place your completed assignment in Box 35.
- 1. Let  $P_1$  and  $P_2$  be two points in  $P_2(\mathbb{R})$ . Clearly  $P_i$  cannot have coordinates (0,0) with respect to any ordered basis for the line  $P_1P_2$ . Thus there exists an ordered basis  $B = \{B_1, B_2\}$  for  $P_1P_2$  such that  $P_i$  has coordinates  $(s_i, t_i)$  with respect to B, and such that  $s_1 \neq 0$ .

It follows that  $s_2$  must be some scalar multiple of  $s_1$ , say  $s_2 = ks_1$ .

Prove that  $P_1$  and  $P_2$  are the same point of  $P_2(\mathbb{R})$  if and only if  $\begin{vmatrix} s_1 & t_1 \\ s_2 & t_2 \end{vmatrix} = 0.$ 

- 2. Let  $L_1, L_2$ , and  $L_3$  be the lines x = 8, y = -3, and 3x 7y = 12, respectively, in  $\mathbb{R}^2$ .
  - (a) What are the coordinates of each line?
  - (b) What is the equation for the point of intersection of  $L_1$  and  $L_2$ ?
  - (c) What is the equation for the point of intersection of  $L_1$  and  $L_3$ ?
  - (d) What is the equation for the point of intersection of  $L_2$  and  $L_3$ ?
- 3. Let P = (4, 5), Q = (2, -4), and R = (-3, 17) be points of  $\mathbb{R}^2$ .
  - (a) Are these three points collinear?
  - (b) What are the equation and the coordinates of the line PQ?
  - (c) What are the equation and the coordinates of the line PR?
  - (d) What are the equation and the coordinates of the line QR?
- 4. Let P = (3, 4, 0), Q = (16, 20, -6), and R = (1, 2, 3) be points of  $P_2(\mathbb{R})$ .
  - (a) What are the coordinates of each point, expressed in standard form?
  - (b) Are these points collinear?
  - (c) What are the equation and the coordinates of the line PQ?
- 5. Let  $L_1 = [2, 1, 0], L_2 = [3, 0, 1]$  and  $L_3 = [0, -2, 1]$  be lines of  $P_2(\mathbb{R})$ .
  - (a) Do the three lines intersect in a common point?
  - (b) What is the point of intersection of  $L_1$  and  $L_2$ ?
  - (c) What is the point of intersection of  $L_1$  and  $L_3$ ?
  - (d) What is the point of intersection of  $L_2$  and  $L_3$ ?
- 6. Let  $L_1 = [4, 0, 1]$ ,  $L_2 = [5, 4, 1]$  and  $L_3 = [0, -16, 1]$  be lines of  $P_2(\mathbb{R})$ .
  - (a) Do the three lines intersect in a common point?
  - (b) What is the point of intersection of  $L_1$  and  $L_2$ ?
  - (c) What is the point of intersection of  $L_1$  and  $L_3$ ?
  - (d) What is the point of intersection of  $L_2$  and  $L_3$ ?