PMAT 3331 – Projective Geometry Assignment #6
Winter 2003

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 3:00 pm on March 19, 2003.
- Please place your completed assignment in Box 35.

1. Let P_1, P_2, P_3, and P_4 be distinct collinear points on the line L of $P_2(\mathbb{R})$. Further suppose that each point is in standard form (that is, the last non-zero coordinate is 1). Suppose that $\{B_1, B_2\}$ is an ordered basis for the line L, and that with respect to this basis point P_i has coordinates (s_i, t_i), for $i \in \{1, 2, 3, 4\}$.

   (a) Compute the cross ratio $\text{CR}[P_1P_2, P_3P_4]$ with respect to the basis $\{B_1, B_2\}$.

   (b) For each point P_i, any scalar multiple k_iP_i is an equivalent point in $P_2(\mathbb{R})$, provided that $k_i \neq 0$. Determine the coordinates of k_iP_i with respect to the basis $\{B_1, B_2\}$.

   (c) Compute the cross ratio $\text{CR}[(k_1P_1)(k_2P_2), (k_3P_3)(k_4P_4)]$ with respect to the basis $\{B_1, B_2\}$.

   (d) What conclusion can be made concerning cross ratios?

2. Let P_1, P_2, P_3, and P_4 be distinct collinear points on the line L of $P_2(\mathbb{R})$. Suppose that $\{B_1, B_2\}$ is an ordered basis for the line L, and that with respect to this basis point P_i has coordinates (s_i, t_i), for $i \in \{1, 2, 3, 4\}$.

   Suppose that $\{B_3, B_4\}$ is another ordered basis for L, and that with respect to this basis point B_i has coordinates (α_i, β_i), for $i \in \{1, 2\}$.

   (a) Determine the coordinates of P_1, P_2, P_3, and P_4 with respect to the basis $\{B_3, B_4\}$.

   (b) Prove that the cross ratio $\text{CR}[P_1P_2, P_3P_4]$ with respect to the basis $\{B_1, B_2\}$ is the same as the cross ratio $\text{CR}[P_1P_2, P_3P_4]$ with respect to the basis $\{B_3, B_4\}$.

   (c) What conclusion can be made concerning $\text{CR}[P_1P_2, P_3P_4]$?

3. Consider the line $y = 7x + 1$ and the line $x + 4 = 0$ in $\mathbb{R}^2$. Let L_1 and L_2 denote the respective corresponding lines in $P_2(\mathbb{R})$.

   (a) Express the equations for L_1 and L_2 in homogeneous coordinates.

   (b) At what point of $P_2(\mathbb{R})$ do L_1 and L_2 intersect?

   (c) At what points in $P_2(\mathbb{R})$ does L_1 intersect the reference triangle?

   (d) At what points in $P_2(\mathbb{R})$ does L_2 intersect the reference triangle?

   (e) In what order do the 4 points that we've just found on L_1 occur?

   (f) Sketch the graph of L_1 and L_2 in $P_2(\mathbb{R})$ and identify all points of interest.

4. Sketch in $P_2(\mathbb{R})$ the two lines that correspond to the lines $y = -2x + 5$ and $2y = x - 4$ of $\mathbb{R}^2$. Identify all points of interest.