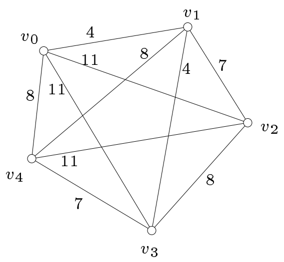
AMAT 3240 – Graph Theory Assignment #7
Fall 1999

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 8:00 am on Friday November 19, 1999.

1. For each $i \in \{0, \ldots, 4\}$, construct a least-weight spanning tree in $G \setminus \{v_i\}$. What can we then conclude about the weight of a least-weight Hamiltonian cycle in G?



2. Given a connected weighted graph G, iteratively remove from G a non-cut-edge of maximum weight until only cut-edges remain. Prove that the resulting subgraph is a spanning tree of minimum weight.

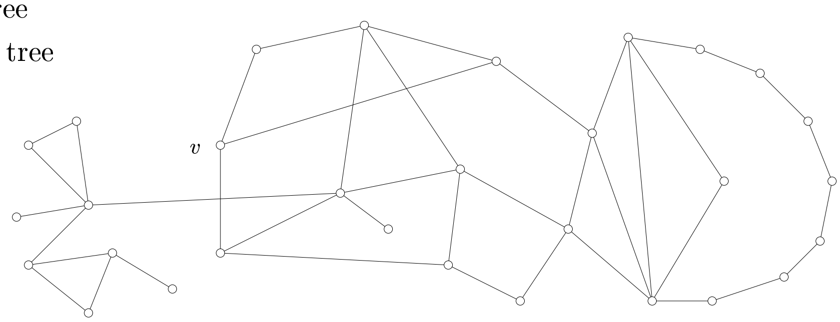
3. Suppose we want to find a minimum-weight spanning path in a connected weighted graph G on n vertices. We have the idea of using a modified version of Kruskal's algorithm: we iteratively select an available edge of least weight so that the edges selected at any given time form a set of paths. When $n-1$ edges have been selected, we have a spanning path in G. Either prove that this algorithm yields a minimum-weight spanning path or else describe an infinite family of graphs for which the algorithm fails to find a minimum-weight spanning path.

4. In the following graph, find

   (a) a depth-first search tree

   (b) a breadth-first search tree

   both starting at v.



5. Suppose we modify algorithm 12.5 of the text, with the only change being that $\mathcal{T}$ is now a tree produced by a breadth-first search in G, instead of a depth-first search. Either prove that the modified algorithm always yields a strongly connected orientation or else find a graph (or, even better, an infinite family of graphs) for which it does not.

6. Suppose that we have built a breadth-first search tree in a connected graph G and each vertex v now has an associated depth $d(v)$. Prove that if uv is an edge of $E(G)$, then $|d(u)-d(v)| \leq 1$.